Erratum - baseline magnetic field oscillations: possible SIM effects on solar irradiance and temperature at Earth

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ABSTRACT

In this erratum paper we reenforce our findings of the millennial oscillations (or Hallstatt cycle) of the baseline solar magnetic field, solar irradiance and baseline terrestrial temperature detected from Principal Component Analysis of the observed solar background magnetic field in the original paper.1 The period of baseline magnetic field oscillations can be expanded to 2200-2300 if the quadruple and sextuple magnetic waves are added. We support the existence of these millennial oscillations with the new references to the similar oscillations detected in carbon $^{14}$C isotope abundances, with wavelet analysis of solar irradiance in the past 12 millennia indicating the presence of this period among a few others. We also support the idea expressed in the original paper that solar inertial motion (SIM) can cause these millennial magnetic field variations because of a change of the distance between the Sun and Earth. In this paper we correct the S-E distance used in estimations in the last section of our paper with the distance derived from the current JPL ephemeris, so that the difference between the closest and furthest S-E distance to be reduced to 0.0027 au per 1000 years. We present estimations demonstrating that this difference in the Sun-Earth distance caused by SIM combined with the variations of solar activity are accountable for the observed variations of the total solar irradiance (TSI) and the baseline terrestrial temperature variations since Maunder Minimum. These estimations also show that the Sun will still continue moving towards the Earth in the next 700 years that will result in the further increase of the baseline terrestrial temperature by to 2.5-3.0°C in 2700 as it was stated in the original paper.1 These variations of solar irradiance will be over-imposed by the variations of terrestrial processes, solar activity of 11 cycles and two grand solar minima to occur in 2020-2053 and 2370-2415 caused by the double dynamo action inside the Sun.2

1 Baseline magnetic field oscillations and solar inertial motion

In the original paper1 we reported the summary curve of two principal components of solar background magnetic field measured from the full disk magnetograms at Wilcox Solar Observatory, US, as a new proxy of solar activity versus the averaged sunspot numbers as suggested by Zharkova et al.2 In Fig.1 we present 3000 years of the summary curve (blue curve) calculated backward from the current date using the method proposed,3 on which we overplotted the graph of the averaged sunspot numbers restored from the dendrochronologically dated radiocarbon concentrations derived by Solanki et al.3 (red curve). The solar irradiance curve prior 17 century was restored from the carbon isotope $\Delta^{14}$C abundances in the terrestrial biomass merged in 17 century till present days with the solar activity curve derived from the observed sunspot numbers.

The summary curve shows accurately the recent grand minimum (Maunder Minimum) (1645-1715), the other grand minima: Wolf minimum (1300-1350), Oort minimum (1000-1050), Homer minimum (800-900 BC); also the Medieval Warm Period (900-1200), the Roman Warm Period (400-150 BC) and so on. These grand minima and grand maxima reveal the presence of a grand cycle of solar activity with a duration of about 350-400 years that is similar to the short term cycles detected in the Antarctic ice.4,5 The 11/22 and 370-400 year cycles were also confirmed in other planets by the spectral analysis of solar and planetary oscillations.6,7 The next Modern grand minimum of solar activity is upon us in 2020-2055.2

1.1 Confirmed: millennial baseline oscillations of magnetic field and solar irradiance - Hallstatt cycle

When the paper by Zharkova et al.1 was written the authors did not realise that this SIM effect has not been considered at all by the models considering conversions of the total solar irradiance (TSI) into the terrestrial temperature. In fact, the SIM was completely ignored. We reported in the previous sections of the paper,1 which constitute more then 97% of the paper contents, the variations of a magnetic field baseline, which also follows the reported variations of the solar irradiance8,9 (shown their
Fig. 6 top plot\(^9\)), and terrestrial temperature.\(^{10}\) The presence of such the millennial oscillations in the terrestrial data is also revealed in the abundances of carbon 14 isotope derived IntCal09 data\(^{11}\) for the last 12,000 years from the trees shown in Fig. 2 (compare the top (our curve) and bottom \((^{14}\text{C} \text{ abundances})\) plots).

For the records, used in Fig.3 from paper\(^1\) the irradiance magenta curve (deliberately scaled down to avoid mess on the plot) was taken from the website http://edsarc.u-strasbg.fr/viz-bin/qcat?J/A+A/531/A6, recommended in the paper.\(^3\) For clarification, this solar irradiance curve restored by Vieira et al.\(^9\) is plotted in Fig.3 (bottom plot) beneath the plot from Fig. 3 from the paper (top plot). It is evident that the red curve in the bottom plot\(^{6,9}\) and magenta curve in the top plot\(^1\) are the same curves at the time of Maunder Minimum. Furthermore, it is evident that the solar irradiance curve in Fig. 3 (bottom plot) demonstrates weak millennial oscillations (red line) over the past 12,000 years with a period of about 2100-2300 years. This is close to that reported in our paper\(^1\) imposed onto some longer-term (16-20K years) orbital oscillations (possibly, one of Milankovich cycles).\(^{12,13}\)

Furthermore, Steinhilber et al.\(^{14}\) combined different \(^{10}\)Be ice core records from Greenland and Antarctica with the global \(^{14}\text{C} \text{ tree ring record using principal component analysis to derive total solar irradiance used as a proxy of solar activity. Steinhilber et al.\(^{14}\) run also the wavelet analysis and confirmed among other periods the periodicities in solar irradiance of 2200 years, or Hallstatt cycle (their Fig.4), which had not been explained yet until now. Curiously enough, the largest amplitudes, or maxima, of Hallstatt cycle minima\(^{14}\) are found centred at approximately 8,200; 5,500; 2,500; and 500 years that closely corresponds to the maxima derived in the baseline (blue) curve in Fig. 2, top plot. Furthermore, the maximum at 500 year reported by Steinhilber et al.\(^{14}\) implies that the minimum of this current Hallstatt cycle should occur close to Maunder minimum similar as we also shown in Fig. 3 (top plot).

The period of 2100 years detected for baseline magnetic field oscillation in the original paper\(^1\) using the first two (principal) components of magnetic dynamo waves produced by dipole magnetic sources, can be expanded to 2200-2300 years if the further magnetic waves produced by quadruple and sextuple magnetic sources\(^{15,16}\) are added.

**The point 1:** Our paper\(^1\) has correctly showed that the baseline magnetic field has millennial oscillations with a period of about 2100 years (their Figs.1-2) close to Hallstatt period (2200) reported by other authors\(^{14}\) also confirming that the current period started from Maunder minimum (their Fig. 3). In this section of the current paper we reconfirm these baseline millennial oscillations also with the data for the terrestrial \(^{14}\text{C} \text{ abundance,}^{11}\) with visual solar irradiance variations\(^{8,9,14}\) and with the spectral analysis using wavelet transform performed on the solar irradiance restored for the past 12 millennia.\(^{14}\)

### 1.2 Proposed qualitative interpretation of millennial periodicities

In order to explain the millennial periodicities of magnetic field baseline and solar irradiance discovered in our paper\(^1\) and explained in the section above, we could not find any other reason rather than the solar inertial motion (SIM) to be the cause of these oscillations. This can occur because during SIM the distance between the Sun and Earth is also changed on a millennial scale, in addition to the seasonal changes. The rough estimations of these variations of a distance between the Sun and Earth were based on the published distances of SIM motion\(^{17,18}\) claiming the diameter of these SIM variations to be about \(4.3R_{\odot} = 0.02 \text{ au, where } R_{\odot} = 695510 \text{ km is the Sun’s radius. We also used the information on the website of the NASA for exoplanet search}\ https://spaceplace.nasa.gov/barycenter/en/\ claiming the wobbling star motion about the barycentre is used to discover stars with the planetary systems.

What we did in the last section of the paper\(^1\) is to provide some general comments about possible solar irradiance variations at different phases of the Earth orbital rotation for different hemispheres if this Sun-Earth distance would change in time of the Sun’s position in the SIM. We did not produce any plots or formulas to support our suggestions, it was just a general idea At that time we were not sure if these SIM distance and irradiance changes are too large or too small, we just put forward the idea that the SIM effect can cause the variations of solar magnetic field baseline and irradiance. Obviously, even mentioning this idea was considered the serious threat by AGW people, who made the Editor of SR to retract our original paper\(^1\) on some unsubstantiated grounds.

**Point 2:** In the last section in our paper\(^1\) we gave only qualitative explanation how the SIM could affect the solar irradiance at the Earth in different hemisphere because of a change of the distance between the Sun and Earth based on the published estimations of the radius of the SIM. However, we did not link the exact magnitudes of solar irradiance with the terrestrial temperature variations. These explanations do not change any of the other results shown in Figs. 1-3 of our paper\(^1\) because they were obtained independently from any suggestions of their origin but from the magnetic field observations of the Sun.

### 1.3 The Sun-Earth (S-E) distance based on the JPL ephemeris

After our paper\(^1\) was published some critique occurred about the hinted S-E distance d changes (up to 0.02 au, or \(0 < d < 0.02\) ) caused by SIM. The critics came from the modellers of the terrestrial temperature variations, who did not even know about or
did not consider SIM in their models at all. They claimed that there are no any changes of the S-E distance with time.

In order to resolve this issue, we have looked at the calculations of the Earth JPL ephemeris by Folkner et al.\textsuperscript{19} presented in the website Alcyone \url{http://www.alcyone.de/aegis/jf/junctions.html} including the effects of four large planets (Jupiter, Saturn, Neptune and Uranus) based on the averaged integration of motion equations for distant planets. We wish to point out to readers that after we submitted the second reply to the Editorial Board in September 2019, in October 2019 this option to include all four large planets for calculation of the Sun-Earth distance in the JPL ephemeris has been removed from this software Alcyone.

The current S-E distance variations vs time derived in the past 200 years (from 1800 to 2000) from the JPL ephemeris\textsuperscript{19} are presented in Fig.4. This plot demonstrates without doubts that the SIM variations are also expanded onto the Earth orbit that confirms the variations of the S-E distance to still occur, only on a slightly smaller scale that the Sun’s SIM (see Fig.4. One can see that, according to the existing JPL ephemeris, the Sun-Earth distance is currently decreasing, as we suggested (being within the limits of 0 and 0.02 au), thus not violating any of our statements in the paper. The current reduction of the S-E distance happens with the average rate of 0.00027 au per 100 years (compare the y-axis readings in Fig. 4 between any 1900 and 2000 years), or by 0.0027 au, or 403914 km, per 1000 years (from 1700 to 2700.)

We also found that if the gravitational effects of large planets on the Earth orbit are calculated by the direct integration of motion equations, as carried out by Laskar et al.\textsuperscript{20–22} instead of averaged integration of motion equations used by the current JPL ephemeris,\textsuperscript{19} this distance decrease between the Sun and Earth can be grown by up to a factor 1.5 to 0.004 au, or 605871 km (see Fig. 4 and 5 in Laskar et al.\textsuperscript{20}.

Let us accept that the real distance between the Earth and Sun imposed by large planets is not the one defined by SIM but the one defined by the JPL ephemeris,\textsuperscript{19} as shown in Fig.4, e.g. the S-E distance is changed by either 0.00027 per 100 years, or by 0.0027 au (or 403914 km) per 1000 years. Knowing that 1 au=1.49597871·10\textsuperscript{8} km. Then the distances of Sun to Earth orbit mentioned in the last section of our original paper.\textsuperscript{1} If the Sun’s SIM orbits are shifted closer to perihelion, then the distances would change to 1.467·10\textsuperscript{8} km (instead of 1.471·10\textsuperscript{8} km) at the closest point (perihelion) and to 1.525·10\textsuperscript{8} km (instead of 1.521·10\textsuperscript{8} km) at the most distant point aphelion. If the Sun’s SIM orbits are shifted closer to aphelion, these numbers would change to 1.475·10\textsuperscript{8} km (instead of 1.471·10\textsuperscript{8} km) at the closest point (perihelion) and to 1.517·10\textsuperscript{8} km (instead of 1.521·10\textsuperscript{8} km) at the most distant point aphelion. Hence, the SIM still has the effect on the Earth orbit leading to the change of the Sun–Earth distance as shown in Fig.4 and stated in the last section of our original paper.\textsuperscript{1}

Point 3: Hence, the claim in the last section of the paper\textsuperscript{1} that the distances between the Earth and Sun changed over 1000 years is correct. Although, real magnitudes of the S-E distances should be changed with respect to the JPL ephemeris\textsuperscript{19} depending where the SIM orbits of the Sun are shifted to. If the SIM orbits are shifting closer to the aphelion, then the S-E distance would change at the closest point to 1.467·10\textsuperscript{8} km (instead of 1.471·10\textsuperscript{8} km listed in the paper\textsuperscript{1}) and at the most distant point - to 1.525·10\textsuperscript{8} km (instead of 1.531·10\textsuperscript{8} km listed in the paper\textsuperscript{1}). If the SIM orbits of the Sun are shifting closer to the perihelion (2700-3700), then the S-E distance would change at the closest point to 1.475·10\textsuperscript{8} km (instead of 1.49·10\textsuperscript{8} km listed in the paper\textsuperscript{1}) and at the most distant point - to 1.517·10\textsuperscript{8} km (instead of 1.50·10\textsuperscript{8} km listed in the original paper\textsuperscript{1}). It is evident that the change of the absolute S-E distances does not change the paper\textsuperscript{1} conclusions as we have not yet calculated the solar irradiance in this paper.

### 2 Solar irradiance variations with a change of the S-E distance and solar activity

In this section we prove in numbers that these updated changes of the S-E distance derived from the JPL ephemeris\textsuperscript{19} are sufficient to account for the changes of the solar irradiance at the Earth orbit since Maunder minimum, the idea of which was only suggested in our published paper\textsuperscript{1} to explain the baseline magnetic field variations of the Sun measured at Earth.

#### 2.1 Estimations of solar irradiance with a distance change

A magnitude of the total solar irradiance $S$ variations at the solar-Earth distance $d$ by considering the Sun as a point body emitting radiation with an intensity $I_\\odot$:\textsuperscript{23}

$$S = \frac{I_\\odot}{d^2}. \tag{1}$$

$$I_\\odot = S \cdot d^2. \tag{2}$$

Hence, the irradiance $S$ can vary either because of the variations of intensity $I$ of solar radiation or because of the variation of a distance $d$ between the Sun and Earth. The variations of the solar intensity $I$ is caused by the variations of solar activity induced by the dynamo action in the solar interior discussed in the sections above.

If the intensity $I_\\odot$ of radiation on the Sun is considered to be constant at a given time ($I_\\odot=\text{const}$), then the solar irradiance $S$ can also change because of a variation of the Sun-Earth distance caused by the solar inertial motion. In this case, using the
Below we produce more detailed evaluation of the observed variations of TSI reported by a number of observers and calculations of the expected TSI variations caused by SIM.

2.2 Observed variations of total solar irradiance (TSI) since Maunder Minimum

The restoration of solar irradiance by26,27 shown in Fig. 5 (top plot) demonstrate that during Maunder Minimum the solar irradiance has decreased by 3 W/m². For other authors estimates see Table 1. Note, we do not include in this comparison the most recent restorations of the solar irradiance28-30 considering the re-normalised solar irradiance after Maunder minimum and using a magnetic flux transport model with strongly averaged past solar magnetic field. Although, their predicted increase of the solar irradiance by 1.5 W/m²29 at the present times can fit reasonably well into the solar irradiance variations caused by SIM (see section 2.3).

Reconstruction of the cycle-averaged solar total irradiance back to 1610 suggests an increase of the irradiance by a value of about 3 Wm⁻²26,27 or about 0.22% of the total solar irradiance since the end of the Maunder minimum (see Fig. 5, top plot). There are the solar irradiance measurements varying up to 1370 Wm⁻²25 to 1371Wm⁻²24 or 137231 as measured by NIMBUS 7 instruments. These are, in general, small changes of solar irradiance from the MM till present compared to the hundreds of watts occurring during seasonal and latitude differences, which may have a noticeable impact on the Earth temperature.

Hence, the maximal variations of the solar irradiance measured since the MM are either equal to 1371 – 1363 = 8 W/m²26,27 or to 1370 – 1363 = 7 W/m² reported by Wolff et al.,24 or 1372-1363=9 W/m² reported by Lee et al.31

2.3 Estimated variations of solar irradiance owing to the S-E distance change by SIM

The current estimation of the change of a distance between Sun and Earth by up to 0.0027 au (see Fig. 4) caused by the SIM considering the averaged gravitation of the large planets currently considered in the JPL ephemeris.19 Hence, the Sun will be closer to the Earth at 2700 by a distance of 0.0027 au, then the distance of the Earth to the Sun will be d₂ =1.0 – 0.0027=0.9973 au, its square is ≈ 0.995, giving the solar irradiance at the Earth to be 1363/0.9969 = 1370 for the irradiance of 1363 W/m² restored for Maunder Minimum.26 This is the number comparable with the solar irradiance measured by different authors as reported in the previous section 2.2.

With the current Sun-Earth distance change take from the PL ephemeris (see Fig. 4) the solar irradiance at Earth should be increased towing to the Sun moving closer to Earth in 2700 by 7 W/m² from the averaged magnitude after Maunder minimum (1366) that is 0.51% of the 1363 W/m² magnitude at MM. While by 2100 this increase should be reduced by factor 0.4 (400/1000) and for 2000 by factor 0.3 (300/1000), e.g. the increase of solar irradiance owing to the SIM will be 0.20%, by 2100 and 0.15% by 2000. Although, if the motion equations of the effects of large planets on the Earth orbit are integrated directly as proposed by Laskar et al.20 then this S-E distance can decrease up to a factor 1.5 that can lead to the further increase

<table>
<thead>
<tr>
<th>Authors</th>
<th>Maunder minimum</th>
<th>2000 - 2012</th>
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<tr>
<td>Lean et al., 199521</td>
<td>1363</td>
<td>1366</td>
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<tr>
<td>Steinhilber et al., 201214</td>
<td>1364</td>
<td>1366</td>
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<td>Shirley et al., 199025</td>
<td>-</td>
<td>1370</td>
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<tr>
<td>Wolff and Hickey, 198724</td>
<td>-</td>
<td>1371</td>
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<tr>
<td>Lee III et al., 199531</td>
<td>-</td>
<td>1372</td>
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Table 1. The solar irradiance in W/m² restored and measured since Maunder Minimum

\[ S_1 \cdot d_1^2 = S_2 \cdot d_2^2. \] (3)

Therefore, if at a distance of d₁=1 au the average solar irradiance is 1366 W/m² suggested earlier24,25 then if the S-E distance is changed to d₂ < d₁, (or it is decreased), the solar irradiance S should also change (increase). For example, if the distance d₂ between the Earth and Sun was to be decreased by 0.016 au (as the SIM predicts for the Sun and barycentre) so that it becomes d₂ =1-0.016=0.984 au. Then its square is 0.968256, so that the irradiance of 1367 W/m² divided by the square of the distance becomes 1366/0.968256= 1411.82. The difference in irradiance is 1411.82-1367=44.82 W/m² that is 0.51% of the 1363 W/m².

This explains the number of 3.5% of potential TSI change stated in the first paragraph of the last section of the original paper,1 just for visualisation of the effect.

\[ (400/1000) \text{ and for } 2000 \text{ by factor } 0.3 (300/1000), \text{ e.g. the increase of solar irradiance owing to the SIM will be } 0.20\%, \text{ by } 2100 \text{ and } 0.15\% \text{ by } 2000. \]
of solar irradiance over the 1000 years (from 1700 to 2700) by 0.77%, or 10.5 W/m² by 2700, to 0.31%, or 4.0 W/m² by 2100 and to 0.22%, or 3 W/m² by 2000.

Hence, the simulated increase of solar irradiance since Maunder minimum caused by the S-E distance change derived from the current JPL ephemeris is expected at the magnitudes of (7.0-10.5) W/m² by 2700, and of (2.7-4.0) W/m² by 2100 or (2.0-3.0) W/m² by the present time, where the first numbers refer to the distances taken from the JPL ephemeris by Folkner et al.19 and the second numbers refer to the distances calculated by the method of Laskar.20 The irradiance increase of 2 W/m² is rather close to the magnitude of the TSI increase of 1.5 W/m² (from 1710 to the 2010) reported from the current observations by Krivova et al.29

**Point 4:** The estimated increase of solar irradiance caused by SIM (S-E distance change from the JPL ephemeris) since the Maunder Minimum is comparable or even smaller than the measured variations of the solar irradiance shown in Table 1.

However, if one keeps in mind that the current Hallstatt’s cycle started in 1645 and lasted until 1710 when the TSI was reduced by about 3.0 W/m² as shown by Lean et al26 then a total increase of the solar irradiance since the last grand solar (Maunder) and Hallstatt’s minimum would be the sum of these two magnitudes, e.g. the observed TSI should increase by 2010 by 1.5+3.0=4.5 W/m². Therefore, the restored total solar irradiance should include the combined effect of the variations of solar irradiance S caused by: 1) variations of the radiation intensity I⊙ on the Sun governed by the solar activity itself and 2) the change of the S-E distance d governed by small gravitational effects on Sun and Earth by large planets (SIM). These points will be discussed in more details in section 2.4.

### 2.4 Comparison of the total solar irradiance variations: estimations versus restorations

Let us compare the solar irradiance (SI) variations with that expected from the SI affected by the joint effects of grand solar minimum during MM and by a changing distance between the Sun and Earth as derived from the current JPL ephemeris

1. Solar irradiance S variations at Earth owing to 11 year cycle is about 0.1% of the average magnitude S increasing during maxima and decreasing during minima.31,32

2. Solar irradiance S variations at Earth owing to GSM is about 2.5-3 W/m², or 0.22% of S26,27,33,34 as shown in Fig.5 bottom plot, blue curve). These estimations are also supported by conclusions by Lockwood et al.34,35 showing up to 0.4% contributions of active regions into the solar irradiance intensity I⊙. Since active regions have a limited presence during grand solar minima (GSMs), a larger drop of solar irradiance during GSMS is very likely to expect.

3. Variations of solar irradiance S at Earth owing to the Sun-Earth distance change would be (0.51-0.77)%, or (7-10.5) W/m² of solar irradiance S) per 1000 years until the maximum at 2700 as calculated in section 2 above. This would result in the solar irradiance increase during the past 400 years, after its minimum at MM, by at least 4/10x0.51=0.20%, or 2.7 W/m², by 0.15% or by 2.0 W/m² by 2000 if Folkner et al.19 ephemeris method is used. The magnitudes of TSI increase by 2 W/m² is comparable with the recently reported59 TSI increase up to 1.5 W/m². If the method of Laskar et al20 is applied for the calculations of the Sun and Earth ephemeresis, these numbers of solar irradiance increase from 1700 will be: 0.77%, or 10.5 W/m² by 2700, 0.31%, or 4.2 W/m², by 2100 and 0.22%, or 3.0 W/m², by 2000.

Let us evaluate a long-term increase of the total solar irradiance S from Maunder minimum (MM) to the present time caused by the joint effects described in items 2+3 (of solar activity in grand solar cycle and of a distance change owing to SIM) giving 0.22+0.20%=0.42% of S. This would give the increase of the solar irradiance from the MM until 2100 as follows: 1363x0.0042= 5.7≈6.0 W/m² or until present time (2000) by 0.22+0.15≈0.37%, e.g. 5.0 W/m², from which 3 W/m² comes from the recovery from the GSM and remaining 2 W/m² (close to the reported59 from recent observations from 1710 when GSM already passed) is likely to come from the SIM effect.

Thus, the total irradiance in the present time should be 1363+5=1368 W/m² that is close but slightly smaller than the magnitudes of 1370-1372 W/m² reported from the NIMBUS observations by Shirley et al.25 and Wolff et al.24 Of course, on top of these variations, the regular fluctuations of the solar irradiance with solar cycles of 11 years shown in Fig. 6, bottom plot, blue lines should be added to the estimations above to reflect the real variations of the solar irradiance at Earth at any given time.

**Point 5:** Therefore, the comparison of predicted and measured variations of solar irradiance at Earth since Maunder minimum shows that the solar irradiance is defined by the combined effects of a) the minimal S-E distance variations caused by SIM taken from the current JPL ephemeris19 or the corrected distance change obtained with the approach by Laskar et al.? and b) the variations of solar irradiance caused by solar activity itself. The estimated resulting solar irradiance is well within the limits of the restored and measured solar irradiance shown in Table 1. Although, only SIM can account for the millennial variations, or Hallstatt cycle, of solar magnetic field baseline1 and solar irradiance14 derived from the isotope dating on Earth because the other solar activity variations during the GSM (MM) have different perion of 350-400 years.
3 Effects of solar activity and SIM on the terrestrial temperature

The fact is well established that the solar irradiance induced by the solar activity of 11 years in the past four solar cycles was heading down while the terrestrial temperature is increasing as shown in Fig. 6 (bottom plot) taken from the NASA and IPCC report [https://www.ipcc.ch/sr15/]. The blue light curve in the bottom plot reveals the solar activity curves of 11 year cycles and solar irradiance variations (re-normalised) induced during these cycles; the thick blue line shows the average irradiance variations imposed by solar activity of 11 year cycles.

While the estimations of the terrestrial temperature shown in Fig. 6 (top plot from Akasofu\textsuperscript{10} and by the red curve (from NASA web site) in Fig. 6 the bottom plot, reveal a constant growth per century since Maunder minimum until the current times with some natural fluctuations. This temperature increase happens at the same time when the solar activity (blue curve) is decreasing, while approaching the grand solar minimum predicted by Zharkova et al.\textsuperscript{2}

These differences can be reconciled by the joint action of: a) the SIM and solar activity effects discussed in the sections above and b) some terrestrial processes defining the temperature including green house gasses (not discussed in this paper). We concentrated on the item a) and its role in defining some parts of the temperature increase because they must be included before b) and leave the item b) to the specialists in the terrestrial processes.

During the last grand solar minimum, Maunder minimum, the total solar irradiance was reduced by 0.22%\textsuperscript{26, 36} that led to a decrease of the average terrestrial temperature measured mainly in the Northern hemisphere in Europe by 1.0°C shown in Fig. 5 (bottom plot)\textsuperscript{33, 36, 37} caused by various terrestrial processes (volcano eruptions, and sea/ice feedback). This seemingly small decrease of the temperature led to frozen rivers, cold long winters and cold summers.\textsuperscript{36, 38}

Hence, let us assume that this relationship between the solar irradiance difference (0.22%S) and the terrestrial temperature difference (1.0°C) holds for other irradiance variations. Then for the irradiance change of (0.51\textsuperscript{19}-0.77\textsuperscript{20}%) by 2700 caused for the S-E distance reduction because of SIM one needs to add the increase by 0.22% caused by the increase of solar activity itself after the past grand processes (Maunder minimum). This will give the overall increase of solar irradiance by 2700 by (0.73-0.99)%, or by 2100 by (0.42-0.53)%, or (0.37-0.44) % by 2000, where again the first number refers to the distances derived by Folkner et al.\textsuperscript{19} and the second number refers to those by Laskar et al.\textsuperscript{20} With the estimated conversion rate (of 0.5°C of terrestrial temperature per 0.1% of solar irradiance as discussed a paragraph above) this increase of solar irradiance would lead to the baseline temperature increase since MM by (3.7- 5.0)°C in 2700, or about (2.1-2.7)°C in 2100, and by (1.9-2.2)°C in the present time (2000-2020).

These numbers, especially the second ones, are close to the baseline temperature variations shown by Akasofu\textsuperscript{10} in Fig. 6 (top plot) and that used by NASA shown in Fig. 6 (bottom plot). Also the linear increase of the baseline terrestrial temperature (0°.5C per century) reported by Akasofu\textsuperscript{10} follows the similar linear decrease of the averaged S-E distance (0.00025 au per century) derived from JPL ephemeris as shown in Fig.4. The resemblance of the temperature increase with the decrease of the S-E distance indicates that these two processes must be linked.

Although, as indicated in our paper,\textsuperscript{2} in the next 700 (2000-2700) years there are two grand solar minima (GSMs) approaching in the Sun caused by the action of double solar dynamo: the modern GSM1 in 2020-2053 and GSM2 in 2370–2415. These GSMs are expected to cause a decrease of solar irradiance by about 0.22%. This, in turn, can lead to a drop of the terrestrial temperature by up to 1.0°C during MM as per the estimations shown in Fig.5 to reach the temperature during the GSM1 by about 0.9-1.2°C higher than during MM. After the GSM1 is finished, the solar activity will be restored to normal activity as shown in Fig.1 of our paper,\textsuperscript{1} and so does the baseline terrestrial temperature, which is expected to following the temperature variations in line with the phases of solar activity and the effects of SIM.

Point 6: In this erratum paper we demonstrated that the current increase of the baseline magnetic field and solar irradiance are likely caused by the combined effect of a) the change of S-E distance because of SIM and b) the increase of solar activity from Maunder minimum caused by double solar dynamo action. This combined approach can explain the increase of the baseline terrestrial temperature from Maunder minimum by about (2.1-2.7)°C by 2100, or by (1.9-2.2)°C by the present time. when SIM orbits are closer to the aphelion. This will also lead to a further increase of the baseline temperature from 1700 by (3.7-4.9)°C by 2700 (or by further (1.9-2.7)°C from 2000) when the maximum of the current millennial (Hallstatt) cycle is expected because the Sun will be closest to the Earth orbit. After 2700 the Sun will move back to the focus of the ellipse where it supposed to be by Kepler’s law and then closer to the perihelion when the temperature will become decreasing again.

As we stated in the paper\textsuperscript{1} and repeat now, this proposed prediction of the baseline temperature variations does not explain the further temperature fluctuations above the baseline caused by either anthropogenic or other terrestrial activities not considered on our paper.
4 Conclusions

In this erratum paper we confirmed that the millennial oscillations of the baseline of solar magnetic field with a period of 2100 years reported in our paper\(^1\) can be also observed in carbon 14 abundances in the past 12 thousand years and confirmed with the wavelet transform using the restored solar irradiance in the past 12K years these millennial variations as Hallstatt cycle of 2200 years. We suggest that the period of 2100 reported from two magnetic waves induced in the Sun by dipole magnetic sources can be extended to 2200-2300 if magnetic waves produced by quadruple and sextuple magnetic sources are considered.

We shown that these millennial oscillations of the baseline magnetic field and solar irradiance are likely to be caused, according to the JPL ephemeris, by periodic variations of the S-E distance imposed by the solar inertial motion (SIM) about the barycentre of the solar system caused by very small gravitational effects of the four large planets. Currently, the Sun-Earth system is in the SIM phase of decreasing S-E distance that leads to the consequent increase of the baseline magnetic field, solar irradiance and baseline terrestrial temperature.

We corrected the suggested changes of the Sun-Earth distance caused by SIM using the real JPL ephemeris of the Earth rotation about the Sun including the gravitational effects of four large planets., which is found decreasing by 0.00027 au per century or 0.0027 au per 1000 years (from 1700 until 2700).

We also corrected the suggested in paper\(^1\) S-E distances in km of the closest and furthest points between the Earth and Sun given in the last section of our paper\(^1\) with the real distances derived from the JPL ephemeris.\(^{19}\) These corrections allowed us to estimate quantitatively the variations of solar irradiance allegedly caused by the joint effects of SIM and solar activity at different times after MM.

We demonstrated that the smaller S-E distance changes predicted by JPL ephemeris and smaller variations of SI since MM until present times are well within the range of observed variations of solar activity since Maunder minimum for the past 370-400 years derived from the restorations of solar irradiance using the isotope dating method for beryllium \(^{10}\)Be and carbon \(^{14}\)C isotopes.

The estimated variations of solar irradiance for smaller S-E distances do not change the asymmetry of solar inertial motion with respect to different terrestrial hemispheres and qualitative conclusions about the SIM effects on the solar irradiance in these hemispheres as explained in the last section of paper.\(^1\)

The estimations of the growth of solar irradiance caused by the solar activity increase from the Maunder minimum (by 0.22%) and by SIM motion (0.51-0.77% by 2700, or by 0.20-0.31% by 2100, or 0.15-0.22 % by 2000). These variations of TSI can reasonably account for the increase of the baseline terrestrial temperature from the Maunder minimum by about (3.7-4.9)\(^{\circ}\)C (by 2700), (2.1-2.7)\(^{\circ}\)C by 2100, by (1.9-2.2)\(^{\circ}\)C by 2000. This linear decrease of the S-E distance with the rate of 0.00027 au per century supports a linear increase of the terrestrial temperature by 0.5\(^{\circ}\)C per century derived by Akasofu,\(^{10}\) which was used in the original paper.\(^1\)

This TSI and terrestrial temperature increases are expected to be over-imposed by other variations of terrestrial temperatures by the atmospheric processes at Earth and by the solar activity during 11 year cycles and two grand solar minima (GSM1: 2020-2053 and GSM2: 2370-2415)\(^2\) caused by the double dynamo effects inside the Sun.

Hence, in this paper we confirm quantitatively the conclusions of the last section of the original paper\(^1\) that the millennial oscillations of the baseline solar magnetic field are linked with the similar oscillations of solar irradiance (Hallstatt cycle) and of the baseline terrestrial temperature, whose millennial periodicity can be only explained by the small periodic variations of the Sun-Earth distance imposed by the solar system planets, while the magnitude of these variations are also governed by the solar activity effects.
Figure 1. Solar activity prediction backwards 3000 years with a summary curve (blue line) of the two principal components (PCs) of solar background magnetic field (SBMF) versus the reconstruction by Solanki et al. (red line). The summary curve is derived from the full disk synoptic maps of Wilcox Solar Observatory for cycles 21-23, the reconstructed solar activity curve was build by merging the sunspot activity curve (17-21 centuries) and a carbon-dating curve (before the 17 century).
Figure 2. Top plot: the oscillations of the solar magnetic field summary curve (cyan line) calculated backward from 70K to 90K years overplotted by the oscillations of a magnetic field baseline, or its zero line (dark blue line) with a period of about 1950±95 years. The left Y-axis shows the scale of variations of the baseline magnetic field, while the right Y-axis presents the scale of variations of the summary curve. Note, the summary curve here (cyan curve) has a different appearance from that in Fig.1 because it was built with one point per year averaged by 13 Carrington rotations used originally to statistically analyse the full data of 120 thousand years. Bottom plot: The oscillations of the carbon $^{14}$C isotope abundances used by Reimer et al. for solar irradiance dating in the IntCal09 data, which also reveal the Hallstatt cycle period (2100-2200). This period is similar to the one derived by Steinhilber et al. (see their Fig.4) with a wavelet transform from the solar irradiance restored in the past 12,000 years.
Figure 3. Top plot: the close-up view of the oscillations of the baseline magnetic field (dark blue curve) in the current and past millennia with a minimum occurring during Maunder Minimum (MM). The irradiance curve (magenta line) overplotted on the summary curve of magnetic field (light blue curve). Note, the irradiance curve (shown by redline in the bottom plot) is slightly reduced in magnitude for a better view (compare the curve in the bottom plot). The dark rectangle roughly indicates the position of MM coinciding with the minimum of the current baseline curve and the minimum of the solar irradiance. The scale of the baseline variations are shown on the left hand side of Y axis, the scale of the summary curve - on the right hand side.

Bottom plot: unnormalised solar irradiance recovered for the holocene, which demonstrates weak oscillations in the filtered (red) line with a period of about 2100-2300 years, similar to those reported in our paper, which are imposed onto the longer-term (16-20K years) orbital oscillations (possibly, one of Milankovich cycles).
Figure 4. The Sun-Earth distance variations of 0.00025 au per 100 years derived from the current JPL ephemeris\textsuperscript{19} (compare the Y-magnitudes at the times of 1900 and 2000, for example) considering the small gravitational effect of four large planets (Jupiter, Saturn, Neptune and Uranus).
Figure 5. Top plot: variations of solar irradiance since the Maunder Minimum restored by Lean et al. \cite{26} Bottom plot: variations of the terrestrial temperature from Maunder minimum until present time during various cycles solar activity. \cite{31,32,37} Decreases of terrestrial temperature are marked by the blue colour while the increases by the red one.
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References
Figure 6. Top plot: the terrestrial temperature variations as recovery from the ‘mini ice age’ derived by Akasofu. The bottom plot: variations of the terrestrial temperature (red lines) and solar 11 years activity (blue lines) (also taken from https://www.ncdc.noaa.gov).
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