Appendix 1

Variations of Sun-Earth distances in millennium M1: 600–1600 and M2: 1600–2600

The first law of Kepler’s planetary motion states that planets move around the Sun in an elliptical orbits where the Sun is located in one of the foci. In the calculation of Sun-Earth distance the first modern scientific models considered only the gravitational attraction between the Sun and the Earth. In reality, Jupiter, Neptune, Saturn and Uranus modify the Sun’s orbit causing solar inertial motion (SIM).

The charts that comprise this appendix are derived from calculations using VSOP87 (Variations Séculaires des Orbites Planétaires, Solutions de VSOP87, http://neoprogrammics.com/vsop87/planetary_distance_tables/ (Bretagnon and Francou, 1988), to compare the differences between the Sun-Earth distance for the millennium 600 – 1600 AD with 1600 – 2600 AD. VSOP87 was developed and is maintained by scientists at The Bureau des Longitudes in Paris. This institution was founded in 1795 for the improvement of nautical navigation, standardisation of timekeeping, and also astronomical observation. Note that the S-E distances from VSOP87 are close (to 6 decimal digits after coma) to those from the JPL ephemeris reported by JPL ephemeris (Folkner et al., 2014) https://ssd.jpl.nasa.gov/horizons.cgi#top.

The Earth is tilted 23.5° from the vertical to the ecliptics. Since the Earth revolves about the Sun following the Kepler’s first law, the tilt changes the orientation of the Northern and Southern Hemisphere relative to the Sun causing the seasons. The Northern Hemisphere is mostly turned towards the Sun during the summer solstice in June 21, or at ellipse aphelion, while Southern hemisphere is mostly turned towards the Sun during the winter solstice on December 21, or at ellipse perihelion that is applicable for M1 (600 – 1600 AD) (see Figure A1.1). While in M2 (1600–2600) the shortest (local perihelion) and longest (local aphelion) distances are shifting every year from the perihelion and aphelion of the ellipse by up to twenty five days to mid-January and mid-July, respectively (see Figure A1.2). Recall that the velocity in a vicinity of aphelion is slower that the velocity at perihelion. This has important implications for changing the solar irradiance magnitudes incident on the Earth (see Appendix 2).
Figure A1.1. The Sun -Earth distances (Y axis) in au derived from the VSOP87 tool for the years 600 (blue), 1100 (red) and 1600 (green). X axis shows days of a month.

The S-E distances in M1 become decreasing in January-May being higher in 600 AD (blue line) than in January 1600 AD (green line). In M1 the shortest Sun-Earth distance at local perihelion slowly shifts from the December solstice at the year 600 AD towards the end of December, while the longest distance at local aphelion correspondingly shifts from the June solstice towards the end of June.

This shift of the local perihelion and aphelion distances continues in M2 with the local perihelion approaching on 5 January and local aphelion on 5 July in 2020 while they shift further to 15 January and 15 July, respectively, in 2600. In millennium M2 in February-June the Earth-Sun distances progressively decrease for the years 1700 (blue), 2020 (orange) and 2600 (grey) AD while in August-December these distances naturally increase because the Earth moves on the elliptic orbit.
**Figure A1.2.** The Sun-Earth distances (Y-axis, au) derived from the VSOP87 tool for years 1700 (blue), 2020 (red) and 2600 (grey). The X axis shows days of a month.

This year (2020 AD), the Sun was closest to the Earth not at the ellipse perihelion but 15 days later at the local perihelion occurring on 5 January and is furthest from the Earth at the local aphelion on 5 July. In 2600 AD, the Earth will be closest to the Sun (local perihelion) on 15 January, and furthest from the Sun at local aphelion on 15 July.
Figure A1.3. Maximal differences in the Sun-Earth daily distances (Y axis, au) between 600 and 1600 (blue) and between 1600 and 2600 (red). The X axis shows days of a month.

By calculating the maximal differences of the Sun-Earth distances for each day in each month in the two millennia (600–1600 AD and 1600–2600 AD) it becomes evident that the nature of the Sun–Earth distances change between the millennia shifting from classic distances appropriate by Kepler’s laws for elliptic revolution of the Earth about the Sun with the rigid perihelion and aphelion. In particular, the differences of the S-E distances show that in both millennia the Sun shifts towards the part of the Earth orbit corresponding to the spring equinox for Northern hemisphere, and this shift is larger in M2 (1600–2600 AD) than in M1 (600–1600). The Sun-Earth distance decreases in March to April are larger 0.01 au in M2, compared to the change of 0.005 au in M1. This shift of the Sun from the focus of ellipse is caused by SIM.
Figure A1.4. Daily (double) differences (Y axis, au) between the maximal differences in Sun-Earth distances for years 600-1600 and 1700-2600 taken from Figure A1.3. The X axis shows days of a month.

Figure A1.5. Annual variations of the (double) differences (Y-axis, au) between the maximal differences of the Sun-Earth monthly distances in the years 600-1600 and 1600-2600. The X axis shows months of a year.
The curve for double differences shows the Sun-Earth distance with a steeper slope for the period March to June when the Sun is moving closer to the Earth, relative to the period August to December when the Sun-Earth distance is increasing. This indicates that in M1 the decrease of Sun-Earth distances in January-June and increase in August-December are smaller than in M2 that is also clearly observed in Figures A1.3 and A1.4.