

Eigen vectors of solar magnetic field in cycles 21-24 and their links to solar activity indices

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ABSTRACT

Using full disk synoptic maps of solar background magnetic field (SBMF) captured from the Wilcox Solar Observatory for 30 latitudinal bands for cycles 21-24 principal components (PCs), or eigen vectors of magnetic oscillations are obtained. The PCs are shown to come in pairs assigned to magnetic waves produced by dipole, quadruple, sextuple and octuple magnetic sources. The first pair is linked to dipole magnetic waves with their summary curve revealing a reasonable fit to the averaged sunspot numbers in cycles 21-24. This verifies the previous results and confirms the summary curve as additional proxy of solar activity decreasing towards grand solar minimum in cycles 25-27. There is also a noticeable asymmetry in latitudinal distributions of these PCs showing an increased activity in northern hemisphere in odd cycles and in southern hemisphere in even ones similar to the N-S asymmetries observed in sunspots. The second pair of PCs linked to quadruple magnetic sources, has 50% smaller amplitudes than the first, while their summary curve correlate closely with SXR fluxes in solar flares. Flare occurrences are also linked to variations of the next two pairs of eigen vectors, quadruple and sextuple components, revealing additional periodicity of about 2.75-3.1 years similar to observed oscillations in flares. Strong latitudinal asymmetries in quadruple and sextuple components are correlating with the N-S asymmetries of flare occurrences skewed to southern hemisphere in even cycles and to northern hemisphere in odd ones. PCA of solar magnetic field raises perspectives for simultaneous prediction of general and flaring solar activity.

Key words: Sun: activity; (Sun:) sunspots; Sun: flares; magnetic fields; methods: data analysis; methods: miscellaneous

1 INTRODUCTION

Solar activity is currently expressed with the monthly sunspot numbers averaged from many observatories showing regular maxima and minima every 11 years (Schwabe 1843; Wolf 1850a,b) with the leading magnetic polarity of sunspots changing every 11 years giving the full cycle for every 22 years (Hathaway et al. 2002; Hathaway 2015). For the future reference in this paper let us use the averaged sunspot numbers published in the Royal Belgium Observatory (SILSO World Data Center 2021). Prediction of a solar cycle through sunspot numbers has been used for decades as the way of testing the knowledge of mechanisms of the solar

dynamo, e.g. processes providing production, transport and disintegration of the solar magnetic fields.

The solar activity had a modern warming period in cycles 19-21 (Solanki et al. 2004; Usoskin 2008; Usoskin et al. 2008; Solanki & Krivova 2011; Usoskin 2017) when sunspot numbers were growing with a cycle number before cycle 20. Although, the averaged sunspot numbers were becoming lower and lower in cycles 21-24. Now in cycle 25, the solar activity continues to be even lower as can be seen in the steepest increase of spotless days at the start of cycle 25 seen in the past 3 centuries as shown in Fig. 1 taken from the SILSO data/image, Royal Observatory of Belgium, Brussels (SILSO World Data Center 2021). The current solar cycle 25 transition counts already more spotless days than the previous one (currently at 848 vs 817 during the SC23-24 transition) that indicates a start of a deeper solar minimum, possibly a grand solar minimum (Zharkova et al. 2015).

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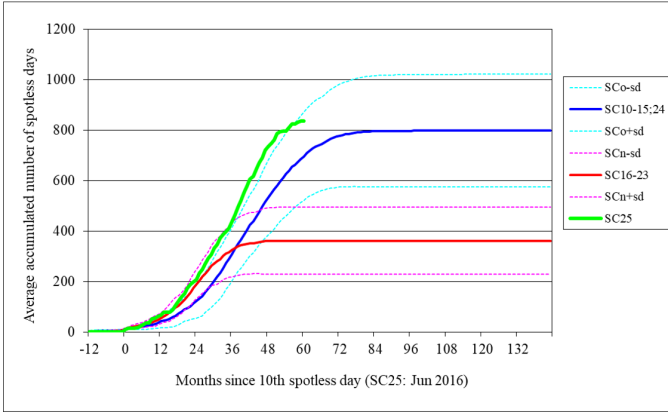


Figure 1. Evolution of spotless days in solar cycles SC24-25 and comparison with the other cycle transits. From the SILSO data/image, Royal Observatory of Belgium, Brussels <https://www.wbis.sidc.be/silso/spotless>.

The thick green line in the plot of Fig. 1 shows the accumulated (total number of) spotless days so far during the transit from solar cycle 24 to 25. The thick blue and red line show the same parameter, but respectively for the average of solar cycles 10 to 15 and 24 (SCo), and of solar cycles 16 to 23 (SCn). This distinction is made because the first group has a long delay (about 4 years) and many spotless days (almost 800 on the average), whereas the other group has barely 2 years between the month with the 10th spotless day and the cycle minimum, and not even half of the total number of spotless day of the first group (less than 400 days!).

Recently, Zharkova et al. (2012, 2015) suggested that instead of the averaged sunspot numbers one can use a new proxy of solar activity derived with principal component analysis (PCA) - the summary curve of two principal components of the solar background magnetic field (SBMF) measured from the Wilcox Solar Observatory (WSO) low-resolution synoptic magnetic maps. The maps are produced in the 30 data points in equal steps of sine latitude from $+14.5/15$ to $-14.5/15$, and for every 5° points of solar longitude range (0-360°). PCA offers a way to extract separate eigen vectors maintained in time reflecting separate physical processes. By averaging the longitudinal data and applying PCA to the magnetic maps in the 30 points of latitude for times of cycles 21-23 (Zharkova et al. 2012), the authors identified the eigen values and eigen vectors, or the periodical magnetic waves, generated in the solar interior by various magnetic sources.

Previously, the PCA was conducted by a few authors (Lawrence et al. 2005) using the NSO/KPNO data containing the full disk synoptic magnetic maps taken with much higher spatial resolution ($1.5''$) than the WSO ones ($20''$). The temporal and latitudinal distributions of solar magnetic field was considered for mean B and median B_M magnetic fields averaged in longitudes. In addition, the authors (Lawrence et al. 2005; Cadavid et al. 2005) did further correction of the NSO butterfly diagrams by using the subtracted magnetic data with 6 CR difference proposed by Benevolenskaya (1995) that makes the magnetic data further randomised.

The results by Lawrence et al. (2005) for the mean field

B were close in some aspects to those derived by Zharkova et al. (2012) having the two principal components, which follow 22 years periodicity of solar cycle, like sunspots. The further investigation of the NSO data was carried out using an independent component analysis (ICA) applied the this difference magnetic data in an attempt to find further fundamental oscillation modes free of noise Cadavid et al. (2005). This approach helped to identify the additional periods of magnetic field oscillations occurring in the solar interior slightly varying from the solar pole to equator (1.3-1.7 years) (Cadavid et al. 2005), besides a 22 year cycle derived with PCA (Lawrence et al. 2005; Zharkova et al. 2012).

Although, the findings by Lawrence et al. (2005); Cadavid et al. (2005, 2008) did not reveal clear pairs of the components and did not find their links to the observable indices of solar activity, like those found by Zharkova et al. (2012). The absence of paired components can be understood in terms of a much higher spatial resolution of the NSO full disk magnetic data, which includes not only the solar background (poloidal) magnetic field but also the (toroidal) magnetic fields of active regions clearly shown in the magnetic 'butterfly diagrams' (Lawrence et al. 2005).

This means that the NSO data entered into the PCA or ICA by Lawrence et al. (2005); Cadavid et al. (2005, 2008) were strongly contaminated by the magnetic field of active regions and sunspots (reaching up to 1000G). While the WSO low-resolution magnetic data have these active region magnetic fields significantly smoothed to match the background field, e.g. a magnetic loop with a magnetic field 1000 G (from NSO with a pixel area approximately $1.''5 \times 1.''5 = 2.''25$, or for simplicity $2.'' \times 2.'' = 4$, or) can be observed in the WSO (pixel area $20.'' \times 20.'' = 400$) as $1000 \text{ G} \times 4/400 = 10 \text{ G}$, or even 5 G for $1.''5$ resolution. Hence, each pixel in the WSO magnetic maps becomes a background magnetic field unlike the NSO containing magnetic fields of active regions.

As result, their PCA produces rather smooth distributions of eigen values in their Spree plots (Lawrence et al. 2005; Cadavid et al. 2005) and does not show clear separation of the eigen vectors into pairs detected in the PCA by Zharkova et al. (2012). These differences restricted the principal component analysis for the NSO data to detecting the background (poloidal) magnetic field near solar poles and the magnetic (toroidal) fields of active region in the royal zone $\pm 35^\circ$ about the solar equator (Lawrence et al. 2005; Cadavid et al. 2005). While the PCA of the WSO magnetic data can be attributed to the solar background magnetic field in the whole solar disk as suggested (Zharkova et al. 2012, 2015).

Each set of the PCs detected from the NSO and WSO magnetic data has it own value for the dynamo modelling while the both data are rather close at higher latitudes emphasising the role of SBFM via polar magnetic field (Benevolenskaya 1996, 1998; Cadavid et al. 2005; Choudhuri et al. 2007). The first pair, or two principal components (PCs) detected by Zharkova et al. (2012) resembling the two first PCs derived from the NSO data (Lawrence et al. 2005; Cadavid et al. 2005), was shown to reflect the primary waves of solar magnetic dynamo with a period of 22 years produced by the dipole magnetic sources (Benevolenskaya 1998; Zharkova et al. 2015). The link of the summary curve of these PCs with the solar activity index of the average sunspot numbers reported by Shepherd et al. (2014); Zharkova et al. (2015) can

be clearly understood because these PCs, in fact, reflect the magnetic butterfly diagrams linked to sunspot occurrences (Lawrence et al. 2005; Cadavid et al. 2005).

The two magnetic waves are found traveling slightly off-phase while moving from one hemisphere to another (Zharkova et al. 2012). Then Shepherd et al. (2014); Zharkova et al. (2015) used the symbolic regression analysis (Schmidt & Lipson 2009) of these two magnetic waves and obtained the analytical expressions describing the magnetic waves incorporated into the solar background magnetic field derived for cycles 21-23 (Zharkova et al. 2012) and attributed to the poloidal field of the solar dynamo (Popova et al. 2013). These mathematical equations were used to make predictions of the two principal components in time, both forward and backward, by a few thousand years and to use them for a comparison with the two-layer solar dynamo model with meridional circulation (Zharkova et al. 2015).

The temporal features of the summary curve of these two PCs, namely its modulus curve used to exclude magnetic polarity, have shown a remarkable resemblance of this curve to the sunspot index of solar activity for cycles 21-23 (Zharkova et al. 2015), despite the summary curve of these PCs reflects the solar poloidal magnetic field while the sunspot index is associated with a toroidal magnetic field of the Sun. This similarity allowed the authors to suggest this summary curve of SBMF as a new solar activity proxy. The advantage of using the summary curve instead of the averaged sunspot numbers is a presence of the extra-parameter, a leading polarity of SBMF, which is shown to be in the anti-phase with the magnetic polarity of leading sunspots (Zharkov et al. 2008).

Using the derived formulae, the summary curve was calculated backwards for 800 years to 1200 revealing the very distinct variations of cycle amplitudes in every 350-400 years, or grand solar cycles. These grand cycles are separated by grand solar minima (GSMs), when the amplitudes of 11 year cycles become very small, similar to those reported in Maunder, Wolf and Oort grand solar minima. The timings of grand solar minima are defined by the interference of two magnetic dynamo waves with close but not equal frequencies (so called beating effect) defined by the different velocities of meridional circulation (Zharkova et al. 2015). The calculation of the summary curve forwards in time until 3200 has also shown the further three grand solar cycles separated by three GSMs with the first GSM to occur in the cycles 25-27, or in 2020-2053 (Zharkova et al. 2015; Zharkova 2020). The various aspects of the criticism (Usoskin & Kovaltsov 2015; Usoskin 2018) of the data used (Zharkova et al. 2015) the derived formulae for two principal components of solar background magnetic field, the use of the summary curve of these PCs as a new proxy for prediction of long-term solar activity, and exploring the role of dipole and quadrupole components (Popova et al. 2018) was clearly answered and the results verified in a few papers (Zharkova et al. 2017, 2018).

There were some objections (Svalgaard & Schatten 2016) to prediction of the upcoming GSM in the next 32 years (Zharkova et al. 2015) and the existence of GSMs in the past, similar to the recorded Oort, Wolf and Maunder grand minima. However, more recent papers verified the GSM existence in the past (Chatzistergos et al. 2017) and confirmed the prediction of the modern GSM in cycles 25-

27 (Kitiashvili 2020; Obridko et al. 2021) by comparing the spectra of zonal harmonics of the SBMF and 3D solar dynamo models using the same WSO synoptic magnetic field data as Zharkova et al. (2015). Furthermore, Velasco Herrera et al. (2021) applied the machine learning (ML) algorithm to the averaged sunspot numbers taken from the International Sunspot Number (SSN) from the World Data Center Sunspot Index and Long-term Solar Observations (SILSO World Data Center 2021) and obtained the similar results reporting the modern Grand Solar Minimum to occur in cycles 25-27, similarly to that reported by Zharkova et al. (2015).

The advantage of using PCA is to separate the eigen vectors defining different physical processes. So far we managed to establish a correspondence of the summary curve of SBMF, produced by the dipole magnetic field, to the production of sunspot numbers defining solar cycles of 11 years (Shepherd et al. 2014; Zharkova et al. 2015). However, the properties of these links are not fully explored yet. There were studies of the periodicity in the N-S asymmetry of sunspot areas from cycles in 1874-1989 (Carbonell et al. 1993; Ravindra et al. 2021) and 1874-1993 (Oliver & Ballester 1994) and 1974-2020 (Leussu et al. 2016; Veronig et al. 2021). The authors reported a well defined North-South asymmetry in all solar cycles and demonstrated with the Monte Carlo modelling (Oliver & Ballester 1994) that the N-S asymmetry cannot be generated by stochastic processes. The similar N-S asymmetry in sunspot numbers was reported for cycles 20, 21 and 22 (Temmer et al. 2002) and cycle 23 (Zharkov et al. 2005). Also there was the N-S asymmetry discovered in the sunspot magnetic fields in cycle 23 derived from the MDI white light and magnetogram full disk images with the automated detection techniques (Zharkov et al. 2005; Zharkov & Zharkova 2006).

In addition, a good correlation is established between the soft X-ray fluxes in flares and the occurrences of H α flares (Vizoso & Ballester 1989; Özgüç et al. 2003; Zharkov & Zharkova 2011) showing short-term oscillations with significant peaks in the power spectrum of H α flare occurrences around 3.1-3.2 years in cycle 22 (Vizoso & Ballester 1989) or around 2,75 years in cycle 23 (Zharkov & Zharkova 2011).

Some authors have made extensive statistical analysis of H α flares, finding its good correlation with a number of solar indices that arise under different physical conditions (Ataç 1989; Ataç & Özgüç 1996; Özgüç et al. 2003). There were also distinct peaks at 26.5 days and 28.3 days reported in the power spectrum of solar radio flux at 10.7 cm for the northern and southern hemispheres (Joshi 1999) and for a few other solar indices (Ataç 1989; Pap et al. 1992; Ataç & Özgüç 1996; Özgüç et al. 2003; Temmer 2021). Zharkov & Zharkova (2011) established a strong links of H α flare occurrences and averaged sunspot numbers and their magnetic field revealing also a clear N-S asymmetry in the H α flare occurrences, e.g. in cycle 22 Özgüç & Altaş (1996) and 23 (Zharkov & Zharkova 2011) The N-S asymmetry was also reported in the soft X-ray flare fluxes in cycles 21-23, which is based on the continuous record of SXR flares observed by GOES (Joshi & Joshi 2004; Joshi et al. 2010, 2015).

The N-S asymmetry observed in the SXR fluxes of solar flares in cycles 21-23 was proven do not occur due to random fluctuations (Joshi & Joshi 2004; Joshi et al. 2010, 2015). It was shown that during the solar cycle 21, the northern hemi-

sphere was more active in production of SXR flux of solar flares but after 1980 it was shifted slightly towards south. In the year 1983 the activity moved towards south strongly and prevailed there during most of the solar cycle 22. In the solar cycle 23, we observe a strong peak in southern hemisphere during the minimum phase in the year 1996 and then the activity is moved towards the north. In the years 1998, 1999 and 2000 the northern hemisphere was dominated. These results are in agreement with the N-S asymmetry of H α flares detected from 1975 to 1999 (Temmer et al. 2001) or from 1996 until 2005 (Zharkov & Zharkova 2011). The preference for northern hemisphere in the data of H α flares during the rising phase of cycle 23 is also reported by Ataç & Özgüç (2001) who also noticed that the behaviour of asymmetry is changed in 2000 and it shifted towards southern hemisphere in the successive years.

So far there were no viable ideas proposed for the explanation of this N-S asymmetry that can be resolved with help of the eigen vectors of solar oscillations derived with PCA. The solar magnetic field oscillations derived with PCA give some positive break through in this problem having detected not only magnetic waves produced by dipole magnetic sources of the Sun's poles but also other magnetic waves produced by quadruple, sextuple or octuple magnetic sources. These additional magnetic waves can be responsible for the shifts of active feature occurrences in the northern hemisphere with respect to the southern one imposed by the appearance of the additional magnetic sources which could be affected by different differential rotation velocities in the opposite hemispheres. For example, Antonucci et al. (1990) first reported in cycles 21 a dominant period of 26.9 days for the northern and 28.1 days for the southern hemisphere, respectively. The similar outcome was confirmed later from the power spectral analysis of daily sunspot numbers during 1975-2000, deriving a rigid rotation with 27.0 days for the northern hemisphere, and 28.2 days for the southern hemisphere (Temmer et al. 2002). These shifts in the differential rotations of northern and southern hemispheres can lead to a redistribution of magnetic sources and formation of quadruple, sextuple and other magnetic sources.

Hence, in order to improve the accuracy of the new solar activity proxy (summary curve) and prediction of solar activity on a millennium timescale (Zharkova et al. 2015), it would be useful to extend PCA to the new data of cycle 24 and to add for consideration of the next pCs of the magnetic waves of the Sun derived from the SBMF, e.g. the next four or six eigen vectors defined by lower variances. The first attempt of such the addition was done by adding theoretical curve for magnetic waves produced by solar dynamo for quadruple magnetic sources (Popova et al. 2018). More beneficial would be to investigate the next pairs of eigen vectors derived from the SBMF with PCA and to explore how their addition is linked to the observations of solar activity feature (sunspots, active regions, solar flares etc.). The resulting magnetic wave obtained from the joint effect of the dipole and quadruple waves can change the visible appearance of solar activity curves and produce some additional features, possibly, not accounted for before.

The new eigen values and vectors derived from the SBMF synoptic maps for cycles 21-24 are described in section 2, the comparison of the summary curves of the first and second pairs and their links with solar activity features are

discussed in section 3, the discussion and conclusions are drawn in section 4.

2 PRINCIPAL COMPONENT ANALYSIS (PCA) OF THE SOLAR BACKGROUND MAGNETIC FIELD

2.1 Derived eigen vectors in cycles 21-24

2.1.1 Brief description of PCA methodology

In order to derive the main periods present in the observational data of solar background magnetic field at different latitudes let us apply Principal Component Analysis (PCA) to the whole set of 2D magnetic field data for cycles 21-24 varying in time and latitude to derive the eigen values and dominant eigen vectors and to compare them with those components derived earlier for cycles 21-23 (Zharkova et al. 2012).

PCA is an orthogonal linear transformation allowing a vector space to be transformed to a new coordinate system, reducing the multi-dimensional data \mathbf{Y} to lower dimensions for better analysis, so that the greatest variance by any projection of the data lies on the first coordinate called the principal component (Jolliffe 2002). The inner (respectively, outer) principal components of a dataset represented by an $(m \times n)$ matrix \mathbf{X} , or so called principal components (PCs), are computed by projecting the original data \mathbf{X} into the eigenspace of the data covariance matrices (Jolliffe 2002).

The standard context for PCA as an exploratory data analysis tool involves a dataset with observations on m numerical variables, for each of n entities or individuals. These data values define $(m \times n)$ -dimensional vectors $Y_{1,}, \dots, Y_m$ or, equivalently, an $(n \times m)$ data matrix \mathbf{Y} , whose j -th column is the vector y_j of observations on the j -th variable. We seek a linear combination of the columns of matrix \mathbf{Y} with a maximum variance.

Such the linear combinations are given by the vector $\mathbf{X} = \sum_{j=1}^m a_j y_j = \mathbf{Y}\mathbf{a}$, where \mathbf{a} is a vector of constants a_1, a_2, \dots, a_m . The variance of such the linear combination is given by $var(\mathbf{Y}\mathbf{a}) = \mathbf{a}^T \mathbf{S}\mathbf{a}$, where \mathbf{S} is the sample covariance matrix associated with the dataset and the sign 'T' denotes 'transposed'.

Hence, identifying the linear combination with maximum variance is equivalent to obtaining a m -dimensional vector \mathbf{a} , which maximises the quadratic form $\mathbf{a}^T \mathbf{S}\mathbf{a}$. For this problem to have a well-defined solution, an additional restriction must be imposed and the most common restriction involves working with unit-norm vectors, i.e. requiring $\mathbf{a}^T \mathbf{a} = 1$.

The problem is equivalent to maximizing $\mathbf{a}^T \mathbf{S}\mathbf{a} - \lambda(\mathbf{a}^T \mathbf{a} - 1)$, where λ is a Lagrange multiplier. This results in the equation defining the eigen values λ as follows:

$$\mathbf{S}\mathbf{a} - \lambda\mathbf{a} = 0. \quad (1)$$

Thus, \mathbf{a} must be a (unit-norm) eigenvectors, and λ the corresponding eigenvalue, of the covariance matrix \mathbf{S} . In particular, we are interested in the largest eigenvalue, λ_1 (and corresponding eigenvector \mathbf{a}_1), since the eigenvalues are the variances of the linear combinations defined by the corresponding eigenvector \mathbf{a} :

$$var(\mathbf{Y}\mathbf{a}) = \mathbf{a}^T \mathbf{S}\mathbf{a} = \lambda \mathbf{a}^T \mathbf{a} = \lambda, \quad (2)$$

Hence, the signs of all loadings (and scores) are arbitrary and only their relative magnitudes and sign patterns are meaningful. Any ($m \times m$) real symmetric matrix, such as a covariance matrix \mathbf{S} , has exactly m real eigenvalues, λ_k ($k = 1, \dots, m$), and their corresponding eigenvectors can be defined to form an orthonormal set of vectors. These linear combinations $\vec{X} = \mathbf{Y}\mathbf{a}_k$ are called the principal components of the dataset.

The principal components (PCs) (*of the dataset variations in time*) are those vectors of \mathbf{X} associated with the largest eigenvalues λ . In practice, we sort the diagonal of λ (the eigenvalues of \mathbf{a}) in a descending order of the magnitude together with the corresponding rows (or columns for the outer PC) of \mathbf{a} (the eigenvectors \mathbf{a}_k of \mathbf{a}).

Principal components are new variables that are constructed as linear combinations or mixtures of the initial variables. These combinations are done in such a way that the new variables (i.e., principal components) are uncorrelated and most of the information within the initial variables is squeezed or compressed into the first components. Geometrically speaking, principal components represent the directions of the data that explain a maximal amount of variance, that is to say, the lines that capture most information of the data. The relationship between variance and information here, is that, the larger the variance carried by a line, the larger the dispersion of the data points along it, and the larger the dispersion along a line, the more the information it has.

For most datasets arising from naturally observed phenomena where the data is noisy, there are comparatively few dominant eigenvalues, which represent the real signal in the data. Thus, this technique simultaneously (i) reduces the data dimensionality, (ii) separates the signals from noise and (iii) orthogonalises the resulting components so that they can be ascribed to separate physical processes.

2.2 Singular value decomposition (SVD) of 2D eigen vectors

In linear algebra, the singular value decomposition (SVD) is a factorization of a real or complex matrix (Golub & Kahhan 1965; Golub & Van Loan 1996). It generalises the eigen decomposition of a square normal matrix with an orthonormal eigen-basis to any ($m \times n$) matrix.

Specifically, the singular value decomposition of an ($m \times n$) complex matrix \mathbf{M} is a factorization of the form $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^*$, where \mathbf{U} is an ($m \times m$) complex unitary matrix, $\mathbf{\Sigma}$ is an ($m \times n$) rectangular diagonal matrix with non-negative real numbers on the diagonal, and \mathbf{V} is an ($n \times n$) complex unitary matrix. If \mathbf{M} is real, \mathbf{U} and \mathbf{V} can also be guaranteed to be real orthogonal matrices. In such the context, the SVD is often denoted as $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^T$, where index 'T' means 'transposed'.

The diagonal entries $\sigma_i = \Sigma_{ii}$ of $\mathbf{\Sigma}$ are known as the singular values of \mathbf{M} . The number of non-zero singular values is equal to the rank of \mathbf{M} . The columns of \mathbf{U} and the columns of \mathbf{V} are called, respectively, the left-singular vectors and right-singular vectors of matrix \mathbf{M} .

We use SVD to separate the 2D matrix $\mathbf{X} = \mathbf{M}$ of solar magnetic data obtained after we applied Principal Component Analysis (PCA) for the 2D magnetic synoptic maps: in latitudes and in time. The eigen values λ_i derived from this

data with PCA are those σ_i used in SVD. In order to separate the temporal eigen vectors and latitudinal eigen vectors, we apply SVD with the given eigen values and obtain the left-singular vectors (dimension of 585) to be temporal ones and the right-singular vectors (dimension of 72) to be latitudinal ones.

2.3 Eigen vectors of solar background magnetic field (SBMF) in cycles 21-24

2.3.1 Two temporal principal components - dipole magnetic sources

In the previous studies (Zharkova et al. 2012, 2015) we utilised the Wilcox Solar Observatory (WSO) 2D synoptic magnetic field data for 30 bands in latitude for a range $\pm 75^\circ$ and in time for cycles 21-23 (Hoeksema 1984), e.g. for Carrington rotations from 1642 to 2026. In the current paper we added the further 2D synoptic magnetic field data for cycle 24, e.g. up to the CR 2246. By running PCA, the updated set of the eigen values of solar magnetic field expressed in the variance assigned to a given component (see section 2.1.1) is obtained as shown in Scree plot in Fig. 2. The eigen values are found equal to those derived previously from the magnetic field of cycles 21-23 (Zharkova et al. 2012) and they still appear in pairs.

The first pair of the eigen vectors, or two principal components (PCs) of the waves, corresponding to the first pair of eigen values, which has the highest percentage of the data coverage by variance (19% and 20%, respectively) covering about 62.4% of the magnetic field data by a standard deviation (Zharkova et al. 2012). This pair of PCs was assigned to those produced by the solar dynamo in two layers of solar interior by dipole magnetic sources (Zharkova et al. 2015). The sum of two PCs is called a summary curve which was shown to define a new solar activity index linked also to the averaged sunspot index (Zharkova et al. 2015).

The next pair of quadruple components QCs covers much less data, 8.6% and 8%, or 16.6%, in total, by variance, respectively, adding extra 11.3 %, achieving 74.6%, in total, by a standard deviation. The quadruple components, QC, are assumed produced by quadruple magnetic sources (Zharkova et al. 2015; Popova et al. 2018). There are further two pairs of eigen values derived in the Scree plot in Fig. 2, which cover the total variance of 13 % for the third pair of sextuple components, SCs, and 10% for the fourth pair of octuple components, OCs. They are covering, respectively, 8.2% and 6.2% of data by standard deviation achieving, consequently, 82.8% and 88.7 %, in total, if all three or four pairs are considered.

The eigen values derived for cycles 21-24 and their relevant eigen vectors, are found to technically undistinguishable from those obtained previously for the synoptic magnetic maps in cycles 21-23 (Zharkova et al. 2012). The two PCs newly derived here for cycles 21-24 are compared with those used earlier for cycles 21-23 (Zharkova et al. 2012) with the result of this comparison plotted in Fig. 3 versus CR numbers (top plot) and calendar years (bottom plot). The new PCs are only slightly shifted along Y-axis for a demonstration of their similarity. It can be clearly seen that the both PC curves for cycles 21-24 and 21-23 have a very close fit, following the same directions and functionalities.

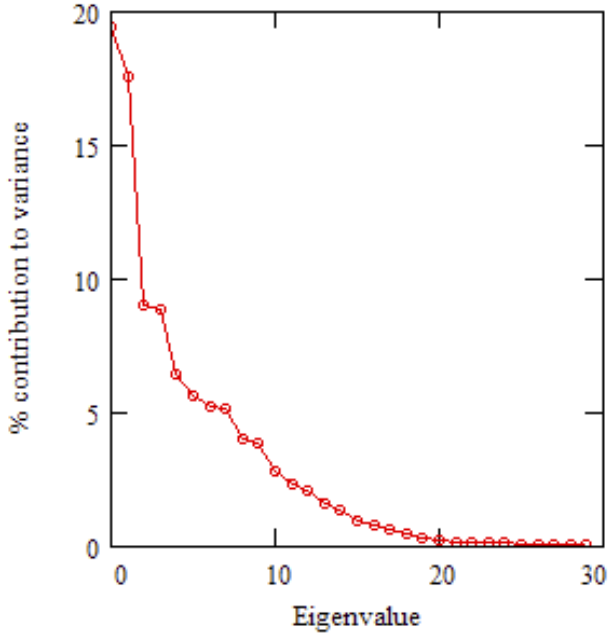


Figure 2. Scree plot: the variance contributions (Y-axis) versus the eigenvalue numbers (X-axis) derived with the PCA decomposition of the solar background magnetic field (SBMF) for solar cycles 21-24 measured from magnetic synoptic maps by the Wilcox Solar Observatory, US (Hoeksema 1984). Note that the Scree plot for the data of 4 cycles 21-24 is identical to that derived previously for 3 cycles 21-23 (Zharkova et al. 2012).

This confirms the previous results based on the magnetic field data for cycles 21-23 (Shepherd et al. 2014; Zharkova et al. 2015) and proves that the eigen values and eigen vectors of solar magnetic field are the intrinsic properties of the waves produced by solar dynamo, which do not change at all or, at least, do not change very quickly within 4 cycles.

The two principal components for dipole magnetic waves derived from the extended data for cycles 21-24 are shown in Fig. 3, bottom plot, revealing the extension of both components into cycle 24. The both principal components (blue and red curves) derived from the historical data in cycles 21-24 show a few clear trends (Zharkova et al. 2012): a) at the beginning of cycle 21 the oscillations marked by the red curve start in the Northern hemisphere while those marked by blue - in the Southern one; b) both waves approach their maxima either in the Northern hemisphere (for odd cycles 21 and 23) or in the Southern one (for even cycles 22, 24); c) the maxima of both waves are overlapping either in the Northern hemisphere (for odd cycles 21 and 23) and in the Southern hemisphere (for even cycles 22, 24) making these hemispheres in a given cycle more active than the opposite one; d) the magnitudes of maxima of the both curves is steadily decreasing by approximately 30-40% each cycle from cycle 21 to cycle 24; d) a phase difference between the components PC1 and PC2 is increasing with a cycle number.

Curiously enough, the variations of the summary curve shown in Fig.4, bottom plot, mimic reasonably close the north-south asymmetry in the average sunspot numbers (SILSO World Data Center 2021) confirming the domination of one (PC1) or another (PC2) magnetic wave at a

given phase of the cycle seen in Fig.3, bottom plot. This N-S asymmetry in sunspot numbers was reported from observations for cycles 20, 21 and 22 (Temmer et al. 2002) and cycle 23 (Zharkov et al. 2005). Also the similar N-S asymmetry was discovered in the sunspot magnetic fields in cycle 23 derived from the full disk MDI white light images and magnetograms (Zharkov et al. 2005; Zharkov & Zharkova 2006). Now these N-S asymmetries can be understood much more clearly, in terms of the reported variations of two eigen vectors of the solar background magnetic field shown in Fig.3 showing the dominant hemisphere where the eigen vectors approach their maxima and, thus, reveal strongest interference.

In order to bring the detected trends in the SBMF closer to the currently-used index of solar activity, the averaged sunspot numbers, we calculated the summary component of the two PCs (the top plot in Fig.4) showing a clear decay of the summary component from cycles 21 to 24 because their amplitudes are reduced. The decay of the summary magnetic wave, representing a poloidal solar magnetic field, is shown to be intrinsic to the mechanism of solar dynamo responsible for generating the two main waves in two different layers of the solar interior (Zharkova et al. 2015). This decay indicates a decrease of the solar background magnetic field from cycles 21 to 24 that was already spotted in observations of the solar magnetic field (Lockwood et al. 2011; Solanki & Krivova 2011) and confirmed with a current decrease of the interplanetary magnetic field the cycle 24 (McComas et al. 2018).

2.3.2 Two temporal PCs - quadruple magnetic sources

The next pair of the two eigen vectors, or quadruple (principle) components (QCs), derived with PCA from the solar background magnetic field are shown in Fig. 5 (top plot). The QC curves are defined by the two eigen values of 8.6% and 8 % of the total variance as per Scree plot (Fig. 2) of the whole solar magnetic field data. By adding these two quadruple (QC) waves to the first two PCs associated with dipole waves, one can improve the accuracy of the data coverage by eigen vectors up to 74.6% by standard deviation. It is evident that the QCs amplitudes (Fig. 5, top plot) are nearly twice smaller than that of the first pair of principal components (PCs) shown in Fig. 4 (top plot). One of the QC is close to the PC3 shown in Fig. 7 in Lawrence et al. (2005).

Furthermore, the temporal variations of QCs are essentially different from those of PCs discussed in section 2.3.1. First, there is no a single maximum per a cycle like it was for the PCs but shown a few maxima into each hemisphere. The blue component seems to have larger amplitudes and more maxima with higher magnitudes in the southern hemisphere (see Fig. 5. top plot).. The strongest maximum of northern polarity occurred between 1986 and 1989 while the two strongest maxima of southern polarity between 1990 and 1993 and 2012-2017. This indicates that the QC waves are more pronounced in the southern hemisphere and this property should be detected in the investigation of the north-south distributions of some solar activity indices different from sunspot numbers.

The summary curve of these two QCs calculated by adding the amplitudes of the components QC1 and QC2 is

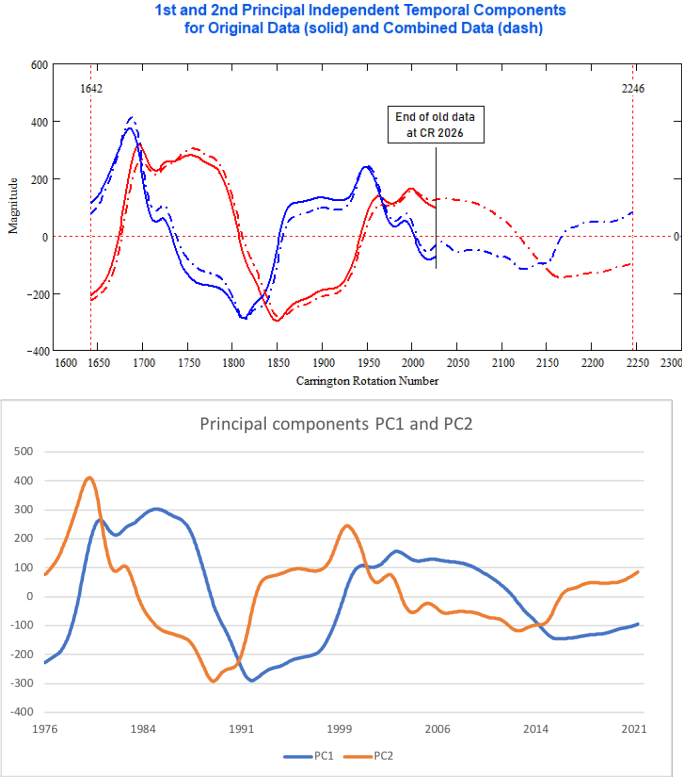


Figure 3. Top: Comparison of the two principal components PC1 and PC2 (Y axis, arbitrary units) derived from the WSO magnetic synoptic maps versus time (X-axis) for the cycles 21-24 (dashed line, the current paper) with those derived previously for cycles 21-23 (solid line) (Zharkova et al. 2012). Bottom: The two principal components PC1 and PC2 (Y-axis, arbitrary units) derived from the WSO magnetic synoptic maps versus time (X-axis) for the cycles 21-24.

plotted in Fig.5 (bottom plot). As result, the summary curve shown in Fig. 5 (bottom plot). shows more maxima into the negative (southern polarity), which have higher amplitudes, even for the recent solar cycle 24, which had much weaker PCs (see Fig. 4). It reveals that the QC waves produced by quadruple magnetic sources is somehow skewed towards the southern hemisphere in cycles 21 and 23 having more extrema with larger amplitudes in the southern hemisphere in comparison with a few occurring in the northern one. The link of this quadruple component with the solar flare features will be discussed in section 3.2.

2.3.3 Two temporal PCs from sextuple and octuple sources

Let us now present the next two pairs of eigen vectors, namely sextuple components SCs and octuple components OCs shown by the two pair of dots below the two dots defining QCs in the Scree plot in Fig. 2. The two sextuple components SCs for cycles 21-24 are presented in Fig. 6 while the octuple components are presented in Fig. 7.

The two principal components for sextuple magnetic sources, SCs, calculated for cycles 21-24 are presented in Fig. 6, top plot and their summary curve is plotted in Fig. 6, bottom plot. It can be observed that the amplitudes of sextuple components SC shown in Fig. 6, top plot are about

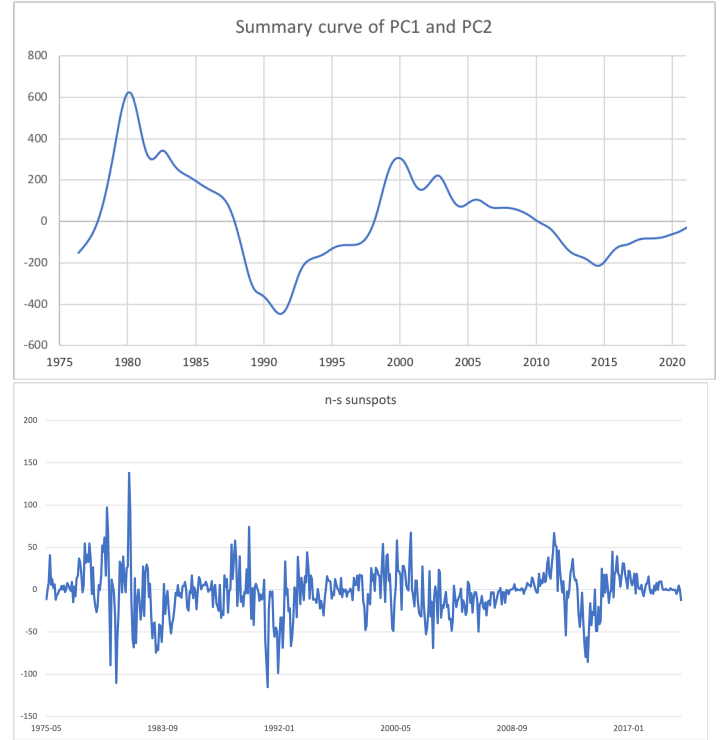


Figure 4. The summary curve of the two principal components PC1 and PC2 from Fig.3 (Y-axis, arbitrary units) versus time (X-axis) for the cycles 21-24 (top plot) and the north-south asymmetry of sunspot numbers (SILSO World Data Center 2021) (Y-axis) versus time (X-axis) for the cycles 21-24 (bottom plot).

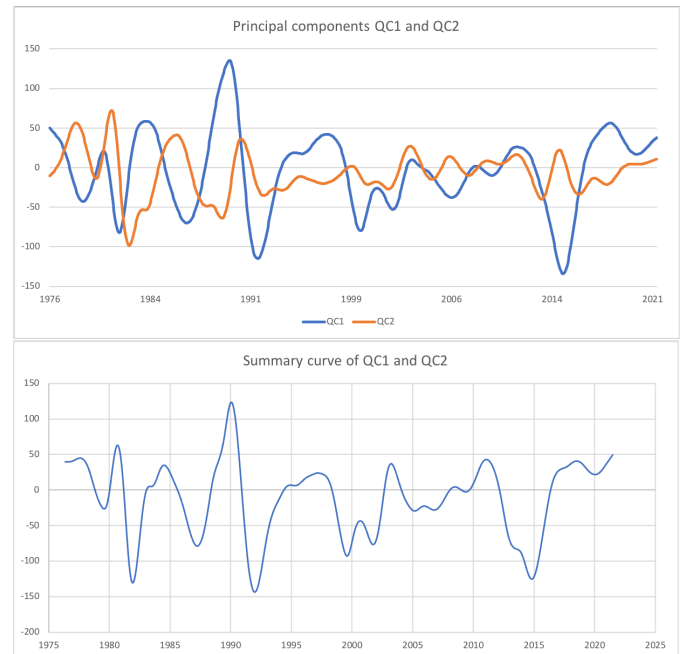


Figure 5. The two quadruple principal components (Y-axis, arbitrary units), QCs, of the SBF data (blue and red lines) (top plot) and their summary curve (bottom plot) plotted versus time (X-axis) in cycles 21-24.

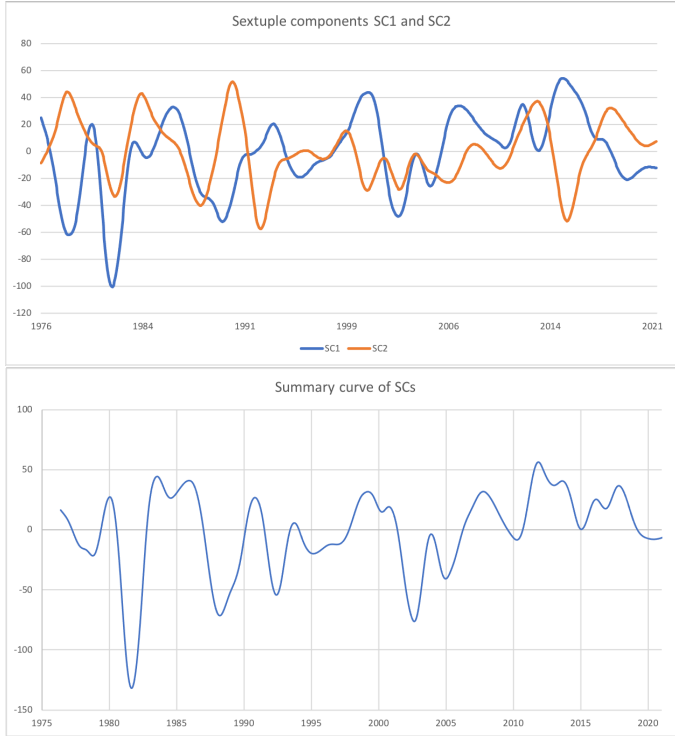


Figure 6. Top plot: the two sextuple principal components (Y-axis, arbitrary units), SCs, of the SBF data (top plot, blue and red lines) and their summary curve (Y-axis) (bottom plot) versus time (X-axis) obtained for cycles 21-24.

20-30% lower than those of the quadruple waves QCs plotted in Fig. 5 because the SC eigen values are also slightly lower than the QC ones. The number of extrema (maxima and minima) in the SCs is essentially different from that of the quadruple waves QCs; and they appear at different times. What becomes evident that the SC components and specifically from their summary curve that towards cycles 23-24 it reveals well defined positive maxima (northern polarity) while large negative maxima occurred for cycle 21. This is different from the quadruple summary curve skewed mainly towards the negative magnitudes (southern polarity) for all cycles.

The next two principal components OCs, assigned to octuple magnetic field sources, calculated from the WSO data for cycles 21-24 are presented in Fig. 7. The amplitudes of OCs are further reduced by about 50% compares to the sextuple SCs revealing a large oscillation in cycle 21 and reduction of these oscillation amplitudes with a cycle number. The oscillations of this OC summary are evenly distributed between positive and negative amplitudes, or northern and southern polarities.

2.3.4 Latitudinal components

Now let us explore the 4 pairs of the latitudinal principal components (LPCs), or eigen vectors, obtained with SCD approach, which are demonstrated in Fig. 8, which are slightly different in the middle to low latitudes from those shown in Fig.9 by Lawrence et al. (2005) since the current WSO latitudinal distributions are not contaminated by the

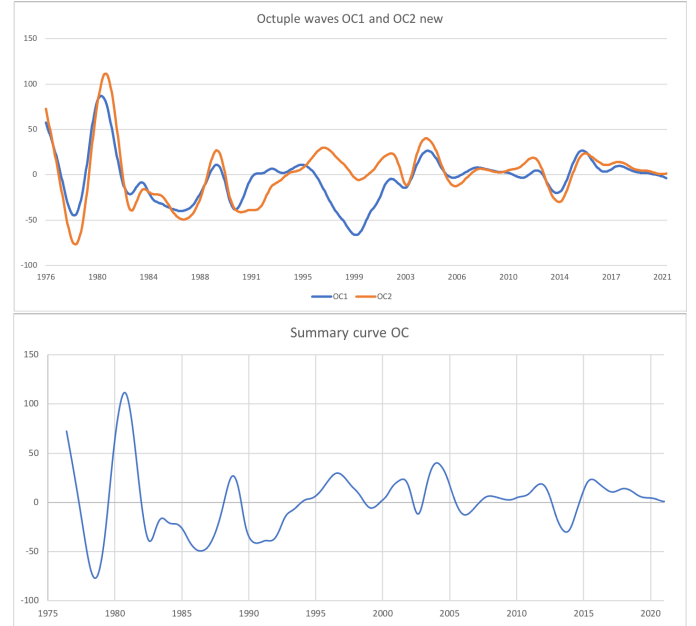


Figure 7. Top plot: the two octuple principal components OCs (Y-axis, arbitrary units, blue and red lines) of the SBF data versus time (X-axis) for cycles 21-24. Bottom plot: The summary curve (Y-axis) of the OCs above (in arbitrary units) versus time for cycles 21-24.

magnetic fields of active regions present in the NSO data (see Introduction). These LPCs are corresponding to the temporal principal components as follows: to two dipole components, PCs, (top left plot), two quadruple components, QC, (top right plot), two sextuple components, SCs, (bottom left plot) and two octuple components, OCs, (bottom right plot).

It can be noted that the two main latitudinal principal components are rather symmetric functions showing the ways how the temporal PCs travel through each hemispheres if the waves are generated by dipole magnetic sources since the pole polarities are to be changed every solar cycle. The dipole LPCs are in the anti-phase in the poles, e.g. they have the opposite magnetic polarities at each pole. The blue curve in latitudinal PC in Fig. 8 (top left plot), (corresponding to the temporal red curve in Fig. 4) starts from the northern hemisphere while the red curve in latitude (corresponding to the blue temporal curve Fig. 4) starts from the southern one.

At higher latitudes beyond $\pm 40^\circ$ the positive wave is in the full anti-phase with the negative one but at lower latitudes the both waves become rather coherent without any phase shift until they reach the latitudes of $\pm 40^\circ$ for the given hemisphere, respectively. In order to achieve this, the positive (negative) magnetic field component passes at about $\pm 40^\circ$ through a wide maximum (minimum) in the Northern hemisphere and a wide minimum (maximum) in the Southern hemisphere, respectively. Then both waves have their local minimum in the Southern hemisphere about -14° for the negative (blue) wave and -18° for the positive (red) one. The latitudes of the positive maxima in the southern hemisphere and the negative minimum in the northern one are close to the latitudes, to which the mean sunspot

locations found from the butterfly diagram approach at the end of solar cycle 23 (Zharkova et al. 2012).

In addition, there are other six latitudinal PCs shown in Fig. 8 corresponding to the lower eigenvalues corresponding to the temporal PCs discussed in sections 2.3.2 and 2.3.3. The next pair of latitudinal waves QCs, supposedly, produced by quadruple magnetic sources (the top right plot in Fig. 8) can help us to understand the propagation of the temporal quadruple waves QCs over the hemispheres. The both (red and blue) QC waves have zero amplitudes near the poles. The blue (negative) wave has the maxima at 45° and -40° latitudes and minimum at the equator, while the red (positive) wave has a maximum about the equator, a minimum at -30° and another maximum at -55° of the southern latitude. These plots clearly indicate that the quadruple latitudinal waves QCs are noticeably skewed towards the southern hemisphere, where the both waves maxima are well defined and higher than those in the northern one.

The opposite happens with the next pair of the latitudinal PCs - sextuple waves SCs (Fig. 8, bottom left plot). The both (blue and red) waves pass through a few extrema. The blue wave has two maxima: one at $+35^\circ$ and another at -10° with the first maximum to be twice higher than the second. While the red wave has nearly equal lower-amplitudes of the two maxima at the latitudes $\pm 25^\circ$, then another maximum at -55° and wide flat maximum at 55° . This indicates that the sextuple waves propagation is skewed towards the northern hemisphere.

For the last pair of latitudinal waves, octuple waves (Fig. 8, bottom right plot), one can observe again the two, positive and negative, waves with four equal amplitude maxima for red wave and four decreasing amplitude maxima for the blue wave. The fluctuations of these two components are in phase having their maxima at latitudes $+50^\circ$, $+15^\circ$, -20° and 55° with the minima in between.

The links of some of these latitudinal waves to the temporal ones and to the observational features of solar activity are discussed below in section 3.

3 LINKS OF THE SUMMARY CURVES TO SOLAR ACTIVITY FEATURES

3.1 Summary curve for dipole PCs and averaged sunspot numbers

Averaged sunspot numbers on the solar disk are considered to be the current solar activity index (Schwabe 1843; Wolf 1850a,b). Most prediction models for solar activity use this index in various attempts to predict its variations for the future solar cycle(s) (see, for example Pesnell 2008, 2020, and references therein). The lack of strongly positive fits of the predicted solar activity expressed in expected sunspot numbers with the real solar activity measured later in sunspot numbers, after the prediction has been made, indicates some consistent disagreement between the variables defining the solar activity as modelled in dynamo models and those measured (averaged sunspot numbers). The dynamo models operate with poloidal and toroidal magnetic fields, with the first one is the SBMF considered above and the second one is related to the magnetic field of sunspots, which is directly linked to the areas covered by sunspots, or an averaged

sunspot numbers. But the latter also contains the number of sunspot groups that clearly deviates from the total magnetic field of sunspots.

Since the SBMF is shown to be in anti-phase with the leading polarity of magnetic field in sunspots (Stix 1976; Zharkov et al. 2008) defining the locations and timing of sunspot appearances on the solar surface and their migration towards the solar equator or poles (Zharkov et al. 2008), one can expect that the summary curve of the SBMF should reveal its well-defined link with the averaged sunspot numbers. The link of the summary curve of these PCs with the solar activity index of the average sunspot numbers reported by Shepherd et al. (2014); Zharkova et al. (2015) can be clearly understood because these PCs, in fact, reflect the magnetic butterfly diagrams linked to sunspot occurrences (Lawrence et al. 2005; Cadavid et al. 2005).

Given the fact that these two entities (SBMF and sunspots) define different magnetic field of solar dynamo action (poloidal and toroidal) defined by different differential equations, and the difference between the solar activity index defining the averaged sunspot numbers from the real toroidal field of the magnetic loops forming sunspots and active regions, one can expect this link should be more causal than identical. Nonetheless, let us now check how the summary curve can be fit for to the current index of solar activity - averaged sunspot numbers. The annual numbers of sunspot were taken from the Solar Influences Data Analysis Center (SIDC) at the Royal Observatory of Belgium (SILSO World Data Center 2021).

By visually comparing the summary curve with the differences between sunspot numbers in northern and southern hemispheres in cycles 21-24 shown in Fig.4 it becomes evident that the sunspot asymmetry loosely follows the summary curve, or the dominant principal component defining this curve. For example, in cycle 21 the dominance of the PC2 with southern polarity at the times close to the cycle maximum coincides with the dominance of sunspot numbers in the southern hemisphere. Shortly later this is changed to a dominance of the PC1 with northern polarity and prevailing numbers of sunspots in the northern hemisphere. The similar tendencies can be seen for other cycles.

In order to link the variations of the summary curve of SBMF to solar activity index using the properties of sunspot and group index expressed in positive numbers, let us calculate, as suggested by Zharkova et al. (2015), the modulus summary curve of the SBMF PCs for cycles 21-24 taken from Fig. 4 by using the absolute values of the negative PC associated with southern polarity of a magnetic field. The overplot of this modulus summary curve onto the averaged sunspot numbers (SILSO World Data Center 2021) for solar cycles 21-24 is presented in Fig. 9 (bottom plot).

One can note a remarkable resemblance between these two curves, given the fact that they represent different magnetic components of solar dynamo waves: poloidal for the modulus summary curve and toroidal for averaged sunspot numbers. The modulus curve follows, in general, the averaged sunspot numbers for all four cycles as it is evident from the plot. In the previous comparison of these curves for cycles 21-23 there were some discrepancies between these curves for the descending phase of cycle 23 (Zharkova et al. 2015), which was later explained by the strongly inflated sunspot numbers used at Locarno observa-

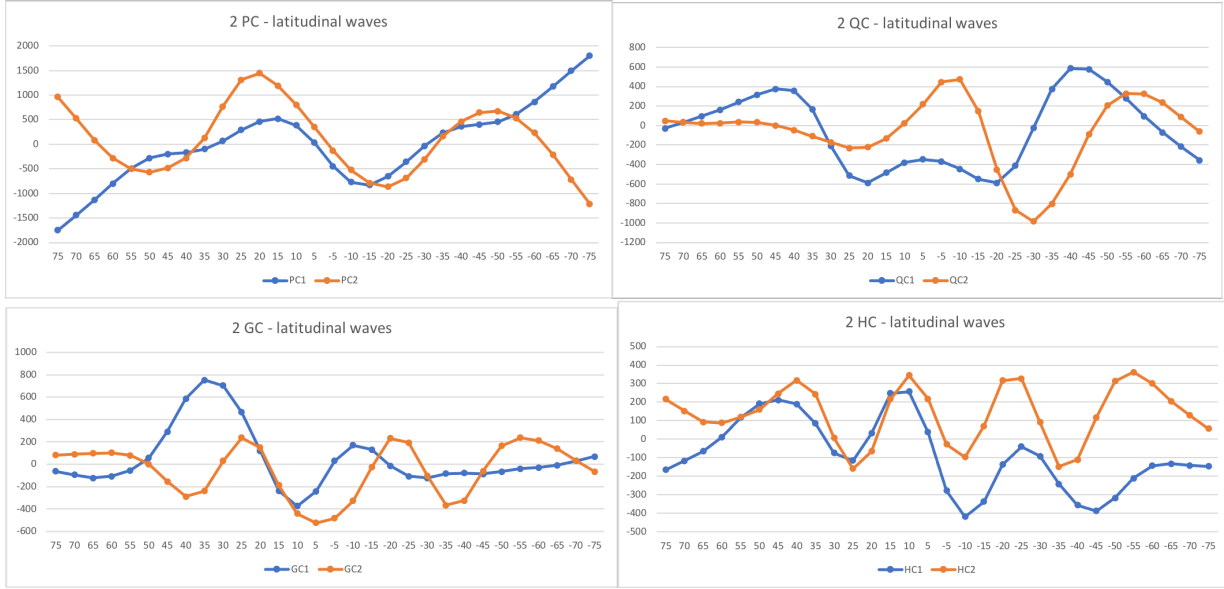


Figure 8. The eight latitudinal principal components (Y-axis, arbitrary units) versus latitudes (X-axis) extracted with SVD from the 2D eigen vectors of the solar background magnetic field obtained with PCA from the WSO magnetic synoptic maps for cycles 21-24.

tory (Clette et al. 2014; Cliver et al. 2015). Now the corrected averaged sunspot numbers fit much better the modulus summary curve in cycle 23 presented in Fig. 9 (bottom plot).

Embracing this similarity between the modulus summary curve (MSC) and averaged sunspot numbers, the MSC can be normalised for each cycle by the averaged sunspot numbers that is now reflected in the left Y-axis of Fig. 9, bottom plot. The modulus curve, in general, follows the averaged sunspot numbers for all the cycles revealing a significant reduction of solar activity from cycle 21 (maximum about 300 sunspots), through cycle 22 (230), 23 (165) to cycle 24 (108). This fits reasonably to the maximum numbers reported for cycles 21-24 (SILSO World Data Center 2021): 21 - 233, 22 - 213, 23 - 180, 24 - 116, keeping in mind that the PCs are derived from the solar background (poloidal) magnetic field while sunspot numbers are related but not equal to the solar toroidal magnetic field.

Hence, from the one hand, the modulus summary curve is found to be a good proxy of the traditional solar activity index contained in the averaged sunspot numbers (SILSO World Data Center 2021). From the other hand, this summary curve is a derivative from the principal components of SBMF with the clear mathematical functionalities representing the real physical process - poloidal field dynamo waves - generated by the solar dynamo at the same time (Popova et al. 2013; Zharkova et al. 2015). This suggestion is supported by the recent research of the same SBMF data from WSO (Kitiashvili 2020; Obridko et al. 2021).

Therefore, the summary curve of the principal components of SBMF evaluated by averaged sunspot numbers and their north-south asymmetries can be considered as a very reasonable proxy of the traditional solar activity understandable by many observers. The summary curve, in fact, represents the eigen vectors of the poloidal magnetic field of

the Sun that can provide a better reproduction of the solar dynamo processes defining solar activity than more artificial index combining sunspot numbers with sunspot groups that hardly can be linked directly to the toroidal magnetic field of the sun. Plus, the summary curve explicitly adds to this proxy the additional parameter - a dominant polarity of the solar background magnetic field for each cycle, which has the polarity opposite to the leading polarity of sunspots (Stix 1976; Zharkov et al. 2008). And this summary curve has a mathematical expression, which can be used for medium and long-term prediction of solar activity useful for many climate studies.

Based on the similarity of the modulus summary and sunspot curves, one can conclude that the solar activity in the cycles 21-24 was systematically decreasing with a cycle number because of the shift in phase of the two magnetic waves so that their phase is increasing approaching an anti-phase when there will be a lack of interaction between these two magnetic waves supposedly produced by solar dynamo. This separation of the two waves into the opposite phases (maximum and minimum) will result in a virtual absence of solar magnetic field on the solar surface, causing, in turn, a lack of magnetic flux tubes appearing on this surface as sunspots. This is likely to lead to a lack of any sunspot activity on the solar surface in cycles 25-27 as predicted by Zharkova et al. (2015), similar to that recorded during the Maunder Minimum in 17 century (Eddy 1976).

3.2 Flare SXR fluxes and quadruple/sextuple magnetic field components

3.2.1 Temporal variations

Let us now explore the role of the quadruple components and their links to solar activity features and, in particular,

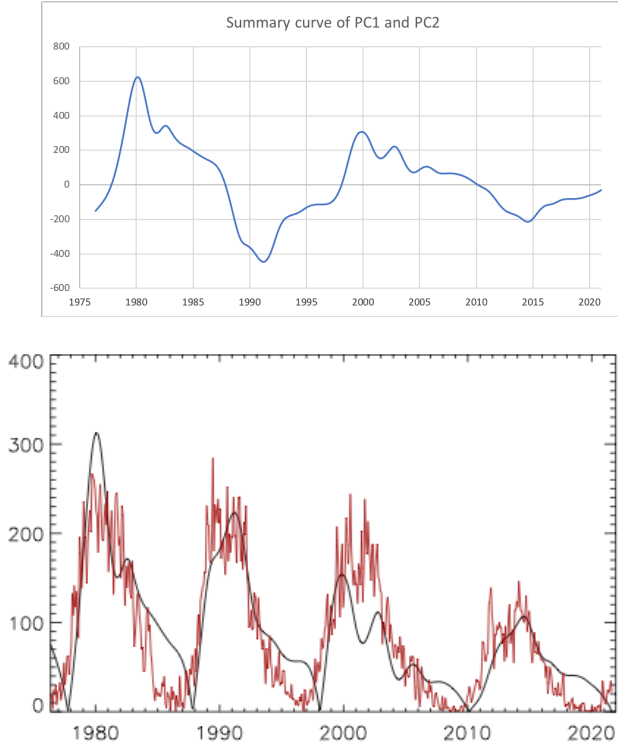


Figure 9. Upper plot: The summary curve (Y axis, arbitrary units) from Fig.4 calculated from the two principal components versus time (X-axis) versus time (X-axis) for cycles 21-24. Bottom plot: The modulus summary curves (arbitrary units) versus time (X-axis) derived from the above summary curve overplotted on the averaged sunspot numbers for cycles 21-24. Y-axis shows the averaged sunspot numbers from the Solar Influences Data Analysis Center (SIDC) (SILSO World Data Center 2021).

the solar activity index in soft X-ray emission flux (FISXR). Previously, Joshi & Joshi (2004); Joshi et al. (2010, 2015) investigated the existence of intermediate term periodicities in the X-ray flare index (F ISXR), which is based on the continuous record of soft X-ray (SXR) flares observed by GOES during solar cycles 21, 22 and part of cycle 23. In this paper we extended this flare data towards the end of cycle 24, considering the SXR flare data locations and energy flux in the time span of 01 June 1976 to 31 August 2017, which was downloaded from NOAA site <https://www.ngdc.noaa.gov/stp/space-weather/solar-data/solar-features/solar-flares/x-rays/goes/xrs/>. To perform the study, we plotted the distribution of the modulus quadruple magnetic field for the period of time of cycles 21-24 shown in Fig. 10, top plot and the time variations of 27-day moving average of the daily soft X-ray (SXR) fluxes for the period 1976-2004 (Joshi & Joshi 2004) is presented in Fig. 10, bottom plot.

It can be seen that the peaks of the SXR fluxes in solar flares occurred in cycles 21-24 are very close to the maxima of the modulus summary curve obtained from the summary curve for quadruple components of the SBMF in Fig. 5. We can compare the three peaks named as 1, 2 and 3 appearing between 1980 and 1983 in the observed SXR fluxes of cycle 21 (Fig. 10 (bottom plot)) with the three maxima in the

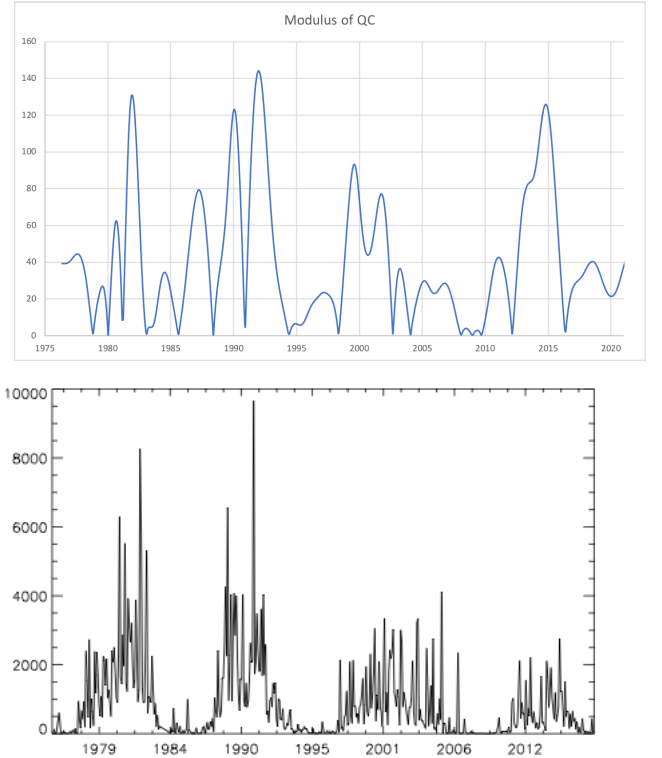


Figure 10. Top plot: The modulus summary curve (Y-axis, arbitrary units) of quadruple components QC of SBMF. Bottom plot: The variations of the SXR emission flux of flares (Y-axis, in units of 10^{-6} W/m²) versus time (X-axis) for the period 1975 - 2017 (see the text for details).

quadruple modulus curve occurred between 1980-1983 in the quadruple field shown in Fig. 10 (top plot). The next cluster of the observed SXR peaks 4 and 5 is seen at about 1988-89 (4) and 1991-1992 (5), which timing nicely fit the two large maxima 4 and 5 in the same years in the modulus curve for cycle 22. This is followed by the gap between 1994 and 1998 followed by the large maximum 6/7 of the flare SXR flux at about 2000-2001 that coincides with the double maxima 6 and 7 of the quadruple modulus curve. The last maximum 8 is observed in the flare SXR flux in 2015-2016 that correspond to the wide tall maximum of the quadruple modulus curve between 2013 and 2017. Therefore, the links of the maxima of the quadruple magnetic field components with the maxima of SHR fluxes of solar flares can indicate a very important role of these quadruple components in the generation of the local flaring activity.

Furthermore, a significant peak of about 3.1-3.2 years was found in the power spectrum performed with the values of flare numbers in the cycles 21-22 (Vizoso & Ballester 1989). Also the sunspot areas and H α flare occurrences in cycle 23 have also shown a periodicity about 2.75 years (Zharkov et al. 2008; Zharkov & Zharkova 2011). To evaluate if this peak is linked the maxima of the quadruple modulus curve, let us count the quadruple wave peaks in the cycles: 21 - 4, 22 - 3, 23 - 4, 24 - 3. Given the averaged cycle duration of 11 years, one can find that the maxima in cycles 21 and 23 occurred every 2.75 years, in cycle 22 and 24 - 3.6 years. The

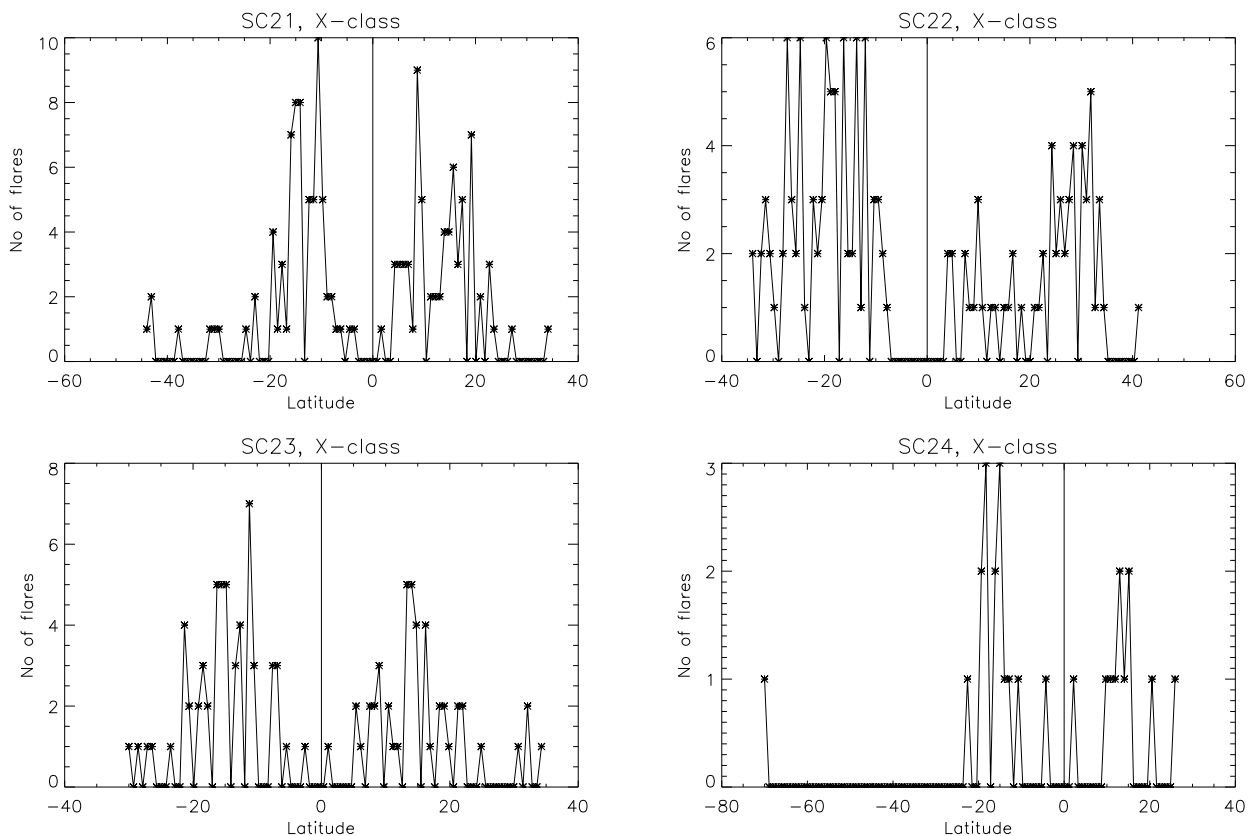


Figure 11. The North-South asymmetry of occurrences of X-class flares in cycles 21 (top left), 22 (top right), 23 (bottom left) and 24 (bottom right).

mean of these periods in cycles 21-24 is equal 3.05 years that is close the flare peak periods in the flare power spectrum reported by Vizoso & Ballester (1989) for cycles 21-22 and is very close to the period of quadruple field variations found in $H\alpha$ flare and sunspot occurrences in cycle 23 (Zharkov & Zharkova 2011).

These similarities in the maxima locations, amplitudes and frequencies between the observed SXR fluxes and quadruple modulus curve indicate that the quadruple component of the solar magnetic field plays a very important role in the initiating the local solar activity in active regions and flares. The quadruple and sextuple magnetic sources can appear because of the shifts of the magnetic sources in the northern and southern hemispheres caused by the differential rotation velocities in the northern and southern hemispheres as measured from the observations (Antonucci et al. 1990; Veronig et al. 2001; Temmer et al. 2002). In fact, by predicting the summary and modulus summary curves of quadruple component of the eigen vectors one can significantly improve the prediction of solar flares and associated with them events, like coronal mass ejections.

3.2.2 Latitudinal variations versus North-South asymmetries

The latitudinal variations of the PCs, or dipole magnetic waves, are rather symmetric in each hemisphere that cannot be said about the other waves generated by different mag-

netic sources. We shown that the quadruple magnetic field (see Fig. 8) reveal larger amplitudes in the southern hemisphere, while the sextuple waves have larger amplitudes in the northern hemisphere. This can be compared to the observational asymmetries indicating that, in addition to the magnetic waves produced by dipole sources, there is domination of sextuple magnetic waves in odd cycles and quadruple waves in even cycles. The joint effects of quadruple and sextuple waves can be detected in the observed N-S asymmetries of the flare index in SXR fluxes.

Following a close link of the quadruple magnetic field to the flaring index in SXR fluxes established in section 3.2, let us now explore the latitudinal variations of magnetic field components, in order to understand the north-south asymmetry often reported in the occurrences of solar flares (Özgülç & Altaş 1996; Joshi & Joshi 2004; Joshi et al. 2010; Zharkov & Zharkova 2011) and sunspots (Temmer et al. 2001; Zharkov et al. 2005). Taking all this into account, here we have made an attempt to investigate the N-S asymmetry of soft X-ray flare index (FISXR) presented by (Joshi & Joshi 2004; Joshi et al. 2010), which is based on the continuous record of SXR flares observed by GOES during solar cycles 21, 22, 23 and 24.

The latitudinal distributions of SXR fluxes from flares of different classes in different solar cycles are presented in Fig. 11-13 and for the whole number of flares for all four cycles are plotted in Fig. 14. The sign 0° represents the equator of the Sun. The variations of N-S asymmetry for

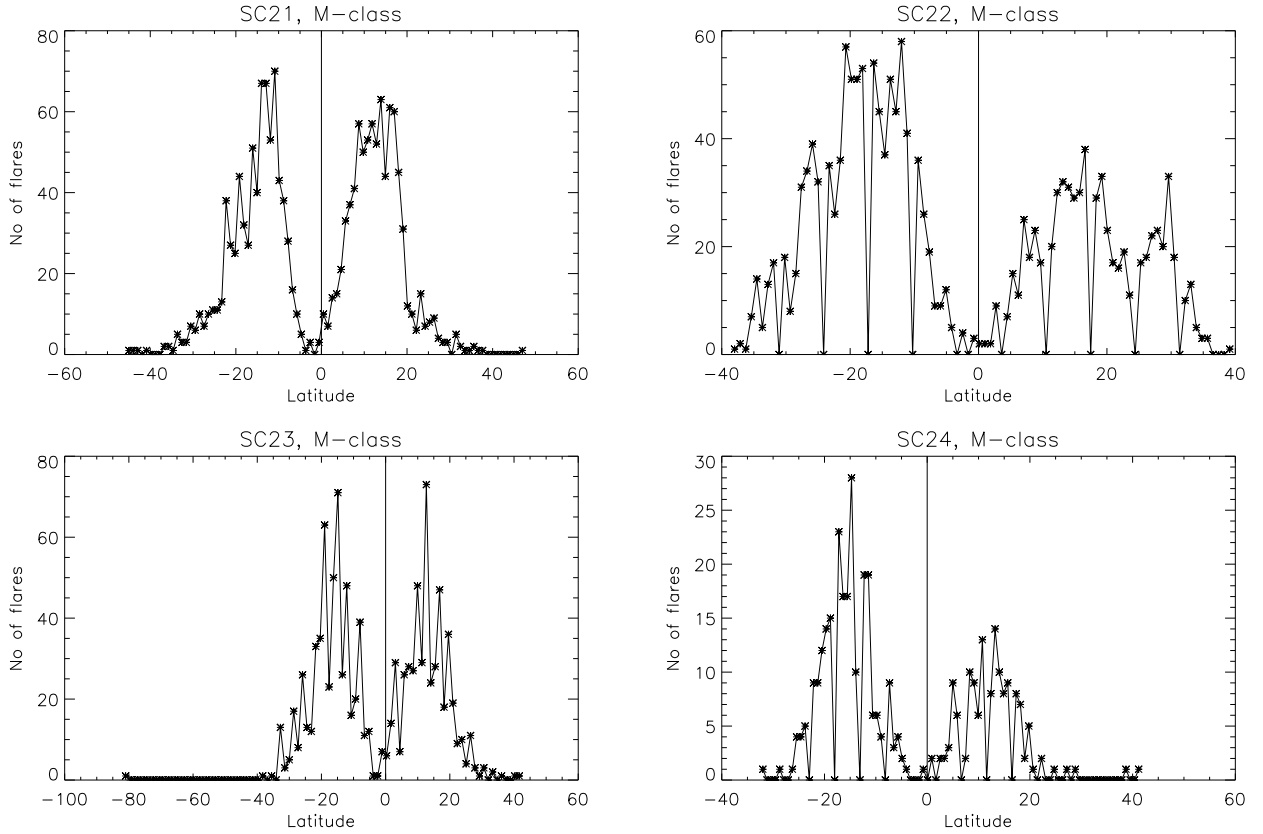


Figure 12. The North-South asymmetry of occurrences of M-class flares in cycles 21 (top left), 22 (top right), 23 (bottom left) and 24 (bottom right).

the total SXR fluxes of flares and for the flares of C, M and X classes reveal the pronounced peaks occurring at 11–20° latitude band on both sides of the solar equator. Although there are consistently higher SXR fluxes in the flares occurring in southern hemisphere for all classes for cycles 21, 22 and 24. Only in cycle 23 for the flares of class M and C the SXR fluxes in both hemispheres have nearly equal numbers.

This latitudinal distribution of number of SXR flares can be interpreted by the latitudinal distributions of principal components, specifically, of the quadruple and sextuple magnetic field components (see Fig. 8, top right and bottom left plots in section 2.3.4) revealing the dominance of quadruple waves in the southern hemisphere and sextuple waves in northern hemispheres (see Fig. 8, bottom left plot). The latitudinal variations of the PCs, or dipole magnetic waves, are rather symmetric in each hemisphere that cannot be said about the other waves generated by different magnetic sources. We shown that the quadruple magnetic field (see Fig. 8) reveal larger amplitudes in the southern hemisphere, while the sextuple waves have larger amplitudes in the northern hemisphere. The latitudinal distributions of SXR fluxes from flares of different classes in different solar cycles presented in Fig. 11-13 and Fig. 14 with evident dominance of southern hemisphere in production of flares with highest SXR fluxes confirm the leading role of quadruple magnetic waves in a formation of flares, since quadruple waves has a dominance in southern hemisphere.

The close magnitudes in latitudinal distributions of

SXR flares with highest fluxes for C-class flares, evidently, indicates that in upper atmospheric layers where X and M class flares occur, the quadruple magnetic components prevail while in lower atmospheric layers where C-class flares and sunspots occur, sextuple components become dominant. This finding lends the support to the different shifts of magnetic sources in the northern and southern hemispheres caused by the differential rotation velocities as measured from the observations (Antonucci et al. 1990; Veronig et al. 2001; Temmer et al. 2002).

Furthermore, temporal variations of the N–S asymmetry indices for the total number of SXR flares with different intensity classes (B, C, M and X) based on the annual SXR flux counts from 1976 to 2007 (see Fig. 15, bottom plot (Joshi et al. 2010)) also reveal the oscillations in latitudes with a dominance of northern hemisphere in odd and southern hemisphere in even cycles. These variations can be compared with the variations of the quadruple and sextuple summary curves presented in Fig. 15, top and middle plots, respectively.

It can be seen from the observational N–S asymmetry of flare occurrences (Fig. 15, bottom plot) that initially during cycle 21, the northern hemisphere was more active but after 1980 it was shifted slightly towards south. In fact, in the year 1983 the activity moved strongly towards south hemisphere and prevailed there during most of the cycle 22 following the propagation of two PCs shown in Fig. 4. In the cycle 23, there is a strong peak in southern hemisphere

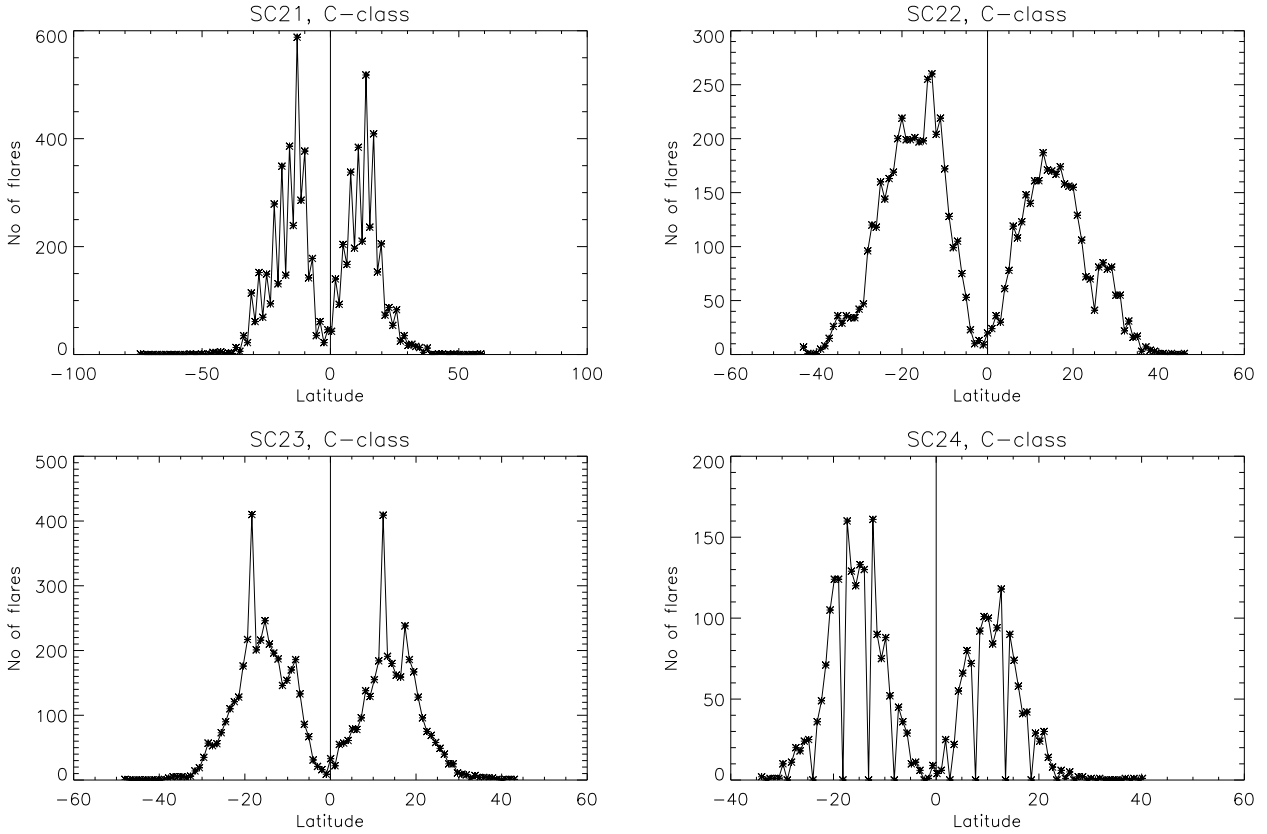


Figure 13. The North-South asymmetry of occurrences of C-class flares in cycles 21 (top left), 22 (top right), 23 (bottom left) and 24 (bottom right).

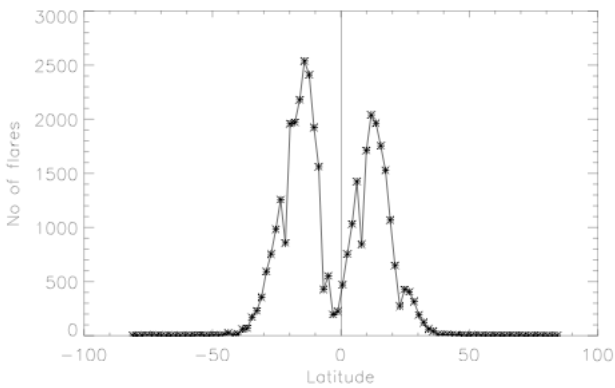


Figure 14. The North-South asymmetry of occurrences of flares of all classes in the period 1975 - 2017 (cycles 21-24).

during the minimum phase in the year 1996 and then the activity is moved towards the north. In the years 1998, 1999 and 2000 the flare occurrence in the northern hemisphere dominated. Then after 2000 there is started a clear domination of the southern hemisphere with a sharp peak in 2006. These results are in agreement with the work done by Temmer et al. (2001); Veronig et al. (2001), who analysed the N-S asymmetry of H α flares from 1975 to 1999, or the N-S asymmetry in H α flares in cycle 23 (Zharkov & Zharkova 2011). The preference for northern hemisphere during the

rising phase of cycle 23 is also reported by Ataç & Özgüç (2001) with the data of H α flares.

By comparing the quadruple and sextuple curves with the observations of the N-S asymmetries of SXR fluxes of solar flares shown in Fig. 15, one can observe the close visual resemblance of the summary curve of the sextuple magnetic field (Fig. 15, middle plot) to the observed curve of the N-S asymmetries plotted in Fig. 15, bottom plot. Combining the plots for the temporal and latitudinal distributions of quadruple and sextuple components of magnetic field waves of SBMF, it is possible to deduce how these waves propagate over the solar latitudes for each cycle and to deduce that the quadruple summary curve dominates the intensity of SXR fluxes in all cycles. While the interaction between quadruple and sextuple waves can define a number of flares occurring in different cycles. Keeping in mind that the quadruple waves in the even cycles 22 and 24 are much stronger than in cycles 21 and 23 (see Fig. 15, top plot) and because of it, the effect of sextuple waves becomes noticeable in the odd cycles. As result, the flare occurrence distributions fluctuate over latitudes becoming skewed towards the southern latitudes in even cycles and northern ones in odd ones.

Hence, the dominance of SXR flares with largest fluxes produced in southern hemisphere and the oscillating numbers of flares produced in northern and southern hemispheres in the odd and even cycles indicate that the magnetic waves from quadruple and sextuple magnetic sources should play the key role in the initiation of a flaring activity

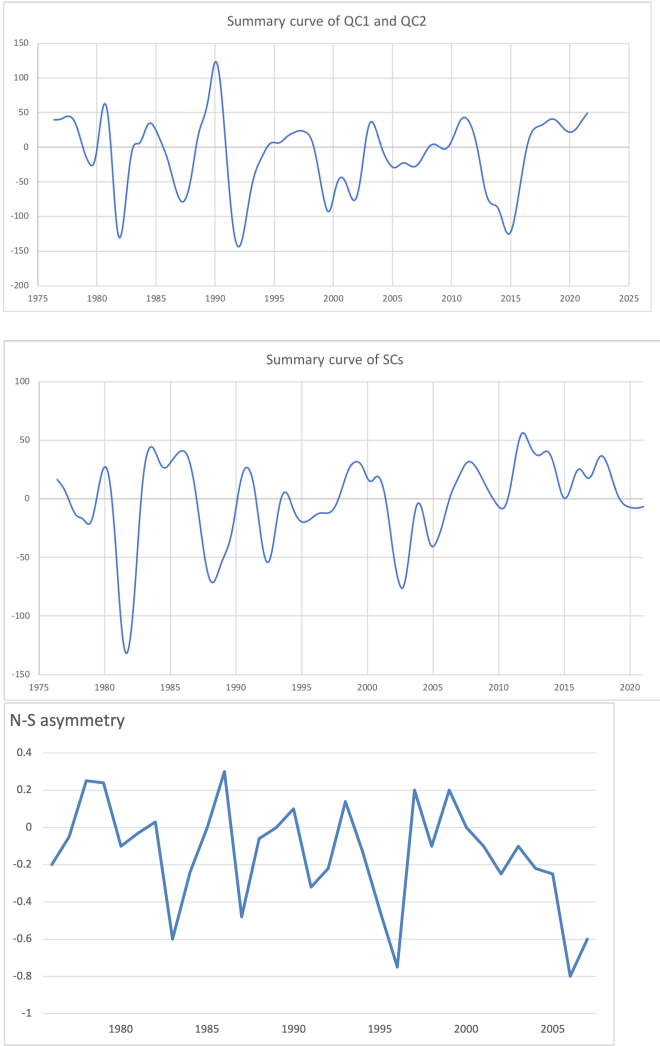


Figure 15. Top: the summary curve (Y-axis, arbitrary units) of quadruple components QC of SBMF with their polarities (positive for northern and negative for southern) versus time (X-axis). Middle: The summary curve (Y-axis, arbitrary units) of sextuple components SC of SBMF versus time (X-axis). Bottom: The North-South asymmetry of monthly SXR flares (Y-axis) for the period 1975 - 2008 (cycles 21-24), similar to the solid curve in Fig.4 by Joshi et al. (2010).

and associated eruptions. The summary curves of quadruple and sextuple magnetic waves generated at different depths by variable differential rotation of the opposite hemispheres can be strong additional factors resulting in more reliable prediction of solar flare occurrences and their maximal SXR fluxes.

4 DISCUSSION AND CONCLUSIONS

4.1 Discussion

In this paper we updated Principle Component Analysis of the synoptic maps of the solar background magnetic field obtained by Wilcox Solar Observatory to four cycles 21-24 and derived the first eight eigen vectors covering the majority of the magnetic data by variance. We confirm our

previous findings of the eigen values and eigen vectors derived for the data obtained for cycles 21-23 and reveal that the eigen values remain the same for the extended data including cycle 24. The eigen vectors of the solar background magnetic field are found to appear in pairs with the total 4 pairs covering more than 88% of the total data variance.

The two PCs newly derived here for cycles 21-24 are compared with those obtained earlier for cycles 21-23 (Zharkova et al. 2012, 2015). The results of this comparison clearly reveal that the both PC curves for cycles 21-24 and 21-23 have a nearly identical fit, following the same directions and functionalities that confirms the previous results derived from the data for cycles 21-23 (Shepherd et al. 2014; Zharkova et al. 2015). The summary curve of the two principal components (PCs) covering 39% of data by variance, assigned to the magnetic waves generated by dipole magnetic sources, shows a steady decrease of the amplitudes from cycles 21 to 24. The decay of the wave amplitudes for both PC waves is intrinsic to the mechanisms of solar dynamo acting in two layers of solar interior responsible for generating the two main waves for the poloidal magnetic field detected as two principal components of SBMF.

The previous PCA and ICA carried out by Lawrence et al. (2005); Cadavid et al. (2005, 2008) using the NSO higher resolution magnetic synoptic maps did not reveal clear pairs of the principal components found by Zharkova et al. (2012). This is believed to happen due to much higher spatial resolution of the NSO full disk magnetic data, which includes not only the solar background (poloidal) magnetic field but also the (toroidal) magnetic fields of active regions clearly shown in the magnetic 'butterfly diagrams' (Lawrence et al. 2005; Cadavid et al. 2005). This means that the NSO data entered into the PCA by Lawrence et al. (2005); Cadavid et al. (2005, 2008) were strongly contaminated by the magnetic field of active regions and sunspots (reaching up to 1000G). While the WSO low-resolution magnetic data have these active region magnetic fields significantly smoothed to match the background field. Each set of PCs (Lawrence et al. 2005; Zharkova et al. 2012) has its own value for the dynamo modelling while the both data become rather close at higher latitudes emphasising a role of SBMF via the polar magnetic field (Benevolenskaya 1996, 1998; Cadavid et al. 2005; Choudhuri et al. 2007; Zharkova et al. 2015).

We also added the further three pairs of the PCs, the waves allegedly generated by quadruple, sextuple and octuple magnetic sources, describing their temporal and latitudinal properties. The two eigen vectors of the quadruple (principle) components (QCs), derived with PCA from the solar background magnetic field covering the other 16.6% of the whole solar magnetic field data by variance. The temporal variations of quadruple and sextuple components are essentially different from those of two PCs produced by dipole sources. QCs amplitudes are found nearly twice smaller than that of the principal components (PCs), the sextuple amplitudes are nearly 30% smaller than quadruple one, and the octuple amplitudes again are 50% smaller than the sextuple ones. The latitudinal variations of the dipole waves are rather symmetric in both hemispheres while the quadruple components are skewed towards southern hemisphere and sextuple waves are skewed more towards northern hemi-

sphere. The octuple reveal again nearly symmetric wave distributions with a few maxima appearing in each hemisphere.

From the one hand, since the SBMF is in anti-phase with the leading polarity of magnetic field in sunspots (Stix 1976) then SBMF defines the locations and timing of sunspot appearances on the solar surface (Zharkov et al. 2008), that, in turn, can explain why the summary curve of the SBMF should have a well-defined link with the averaged sunspot numbers. The modulus summary curve of two PCs from dipole sources for cycles 21-24 was over-plotted onto the averaged sunspot numbers reveals a remarkable resemblance despite the fact that they represent different magnetic components of solar dynamo waves: poloidal for the modulus summary curve and toroidal for averaged sunspot numbers. The modulus curve follows, in general, the averaged sunspot numbers for all four cycles revealing a significant reduction of solar activity from cycle 21 (maximum 300 sunspots), through cycle 22 (230), 23 (165) to cycle 24 (108).

From the other hand, maxima of the modulus summary curve obtained from the summary curve for quadruple components of the SBMF is very close to the peaks of the SXR fluxes in solar flares occurred in cycles 21-24 that can indicate a very important role of the quadruple components in the generation of the local solar activity in active regions. Interestingly, there are regular peaks, or multiple maxima of the quadruple modulus curve, occurring with a period varying between 2.75 years for odd cycles and 3.6 for even ones, or averaged period of 3.05 years, that is close to the periods of 2.75-3.2 years derived from the observational periodicity of flaring activity (Vizoso & Ballester 1989; Zharkov & Zharkova 2011).

The joint effects of quadruple and sextuple waves can be detected in the observed N-S asymmetries of the SXR flare numbers. The dominance of SXR flares with largest fluxes produced in southern hemisphere and oscillating numbers of flares produced in northern and southern hemispheres in the odd and even cycles indicate that the magnetic waves from quadruple and sextuple magnetic sources should play the key role in the initiation of a flaring activity and associated eruptions. These waves can be strong additional factors leading to more reliable prediction of solar flare occurrences and maximal SXR fluxes.

4.2 Summary

In summary, in this paper we managed to:

- (i) Confirm with the data for cycles 21-24 the eigen values and eigen vectors of SBMF derived from cycles 21-23.
- (ii) Explain a difference between the PCA results derived from the low-resolution WSO and high resolution NSO magnetic maps.
- (iii) Confirm a summary curve of two principal components as a new additional proxy of solar activity close to sunspot solar index.
- (iv) Introduce a link of the quadruple pair of PCs to the flare soft-X-ray flux index
- (v) Show the additional three pairs of PCs are linked to biennial (2-3 years) periodicity of flare occurrences for different classes of flares.
- (vi) Link latitudinal variations of different PCs to the

north-south asymmetries observed in sunspots and flare occurrences.

Further investigation is required for the magnetic field components derived with PCA to confirm the prediction of solar activity (Shepherd et al. 2014; Zharkova et al. 2015), to refine the timing of the grand solar minima in the past and the future and to confirm the role of double solar dynamo in generation of these waves that will be a scope of the forthcoming paper.

DATA AVAILABILITY STATEMENT

The data underlying this article will be shared on reasonable request to the corresponding author.

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